Theoretical Modeling of Impulsive Tensile Response of Curved-Fibers Composites

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ABSTRACT

A modeling procedure is proposed to determine impulsive tensile response of a flexible composite of polymer base (resin) and sine-shaped glass fibers. A certain elastic model is adopted and developed in order to determine important mechanical parameters to be used in impulsive response determination in the present procedure.

NOTATION

σ_x, σ_y Stress	In x- and y- axis respectively.
ε_x , ε_y Strain	
τ_{xy} γ_{xy}	Shear– stress and – strain respectively.
L,T	Longitudinal and transverse fiber coordinates respectively.
X,Y	Composite general coordinates.
Θ	Angle between
ε_b	Fiber braking strain.
ε_L	Longitudinal fiber strain in the (L-T) coordinates.
$\bar{\varepsilon}_L$	Transformed longitudinal fiber strain [in the (X-Y) coordinates].
ε^*_L	Averaged fiber-longitudinal strain along curved fiber.
ν^*_{xy}	Effective Poisson's ratio (curved fibers composite).
S_{ij}	Composite compliance in fiber coordinates (L-T).
$\bar{S}_{ij}$	Transformed composite compliance from (L-T) to (X-Y) coordinates.
S^*_{ij}	Averaged compliance for curved-fibers composite.
E_x	Composite longitudinal Young's modulus (straight fibers).
E^*_x	Composite effective longitudinal Young's modulus (curved fibers).
E	Strain energy.
P	Maximum impulsive loading on composite specimen.
W	Falling mass weight in impulsive loading test.
L	Composite specimen length.
A	Composite specimen cross sectional area.
E_y	A constant Young's modulus value.

1. Introduction: Mechanical properties of composites depend basically on mechanical properties of both matrix resin and fibers materials, volumetric ratios of fibers, the status of fibers, i.e. continuous[1], chopped[2], particle like, woven[3] for example. In many applications it is required to develop composites that in addition to be flexible, are capable of withstand high amounts of stresses with considerable resistance to fatigue fracture[4]. These kinds of composites exhibit low stiffness under relatively low stresses and high stiffness and strength under higher applied stresses[4].

Straight-fibers composites cannot be regarded as flexible composites because of the low extensibility of straight fibers in spite of the high ductility of matrix materials that may be used.

A convenient way of achieving flexible composites that can bear high amounts of loadings is to make use of fibers geometrical shape that may by changed in parameters as the loading changes in magnitude.

In this study, sine shaped curved fibers are considered as reinforcing agents of composites of elastomeric polymer matrix that is characterized by high ductility.

The gradual straightening of curved fibers with external tensile loading may be optimized yielding a composite that shows enhanced stiffness with increasing deformation[4].

In this work, an elastic model proposed by Chou and Takahashi[4] is adopted and expanded to generate certain data that are to be used here in developing the impulsive response modeling.

This elastic model is expanded here to determine the Poisson's ratio and strain energy during composite tensile deformation since these two quantities are of importance in further calculations.

In this elastic model, the composite is considered to be exposed to series of strain incremental steps (tensile cycles), after each tensile cycle current (or instantaneous) values of concerned parameters are determined. This process continues until a breaking point is reached indicating the failure of the composite. This failure is regarded when the current tensile strain (ε_x) exceeds a certain breaking value (ε_b).

Then the impulsive tensile loading is dealt with via an approach proposed in this work, where use is made of some data obtained from the elastic model mentioned above.

2. General Overview of the Elastic Model: Iso-phase configuration of the sine-shaped fibers is assumed throughout the composite where they extend along the X-axis as illustrated in Fig.(1). As shown in this figure, fiber coordinates (L,T) and composite coordinates (or general coordinates) (X,Y) make an angle (Θ) with respect to each other at any point throughout the composite (the situation is assumed symmetric along the Z-axis).

The sine-shaped fiber is characterized by it's wavelength (λ) and amplitude (a) as illustrated in Fig.(2).

In this model, coordinates transformation relations (i.e. transformation matrix) [5,6] are employed to achieve transitions between local (L, T)- and general (X, Y)-coordinates for the elastic compliance coefficients, while averaging processes of those elastic compliances along fiber wavelength are performed to represent the curvature of fibers effectively. This yields the averaged strain-stress relation:

$$\begin{Bmatrix} \varepsilon x \\ \varepsilon y \\ \gamma xy \end{Bmatrix} = \begin{bmatrix} S^{*}11 & S^{*}12 & S^{*}16 \\ S^{*}12 & S^{*}22 & S^{*}26 \\ S^{*}16 & S^{*}26 & S^{*}66 \end{bmatrix} \begin{Bmatrix} \sigma x \\ \sigma y \\ \tau xy \end{Bmatrix} \quad \text{—— (1)}$$

where $S^{*}_{ij} = \frac{1}{\lambda}\int_{0}^{\lambda} \overline{S_{ij}}\, dx$ is the averaged compliance of the curved-fibers composite, and $\overline{S_{ij}} \equiv \overline{S_{ij}}\,(\theta)$ is the transformed compliance (S_{ij}) from (L,T) to the (X,Y)-system. Here, the effective Young's modulus for the longitudinal stress-strain response is defined as:

$$E^{*}_{x} = 1 / S^{*}_{11} \quad \text{———— (2)}$$

Coordinates transformation and averaging processes are also employed to obtain an expression for the average strain $\left(\varepsilon^{*}_{L}\right)$ along curved fiber length as:

$$\varepsilon^{*}_{L} = \frac{1}{S}\int_{0}^{S} \overline{\varepsilon_{L}}\, dx \quad \text{———— (3),}$$

here, $\overline{\varepsilon_L} \equiv \overline{\varepsilon_L}(\theta)$, is the longitudinal fiber strain (ε_L) transformed to the (X,Y) coordinate system.

The averaged fiber strain (ε_L^*) is to be compared with a breaking value (ε_b) to check for composite failure.

The calculations of this model provide current values of some important mechanical parameters concerning the composite specimen during a stepwise longitudinal tensile process along x-axes. Among those parameters are: longitudinal stress (σ_x), effective Young's modulus (E_x^*), elastic compliances before averaging (S_{ijs}) and after averaging (S_{ijs}^*), and the average strain along fibers length (ε_L^*).

The calculation process is terminated when ε_L^* exceeds ε_b indicating the failure of the composite.

3. Impulsive Loading: The impulsive loading situation considered is shown in Fig(3), where a certain weight (W) falls through a height (h) along a hinged composite specimen (of curved fibers) of an initial length (L). The falling mass impacts on the tipped end of the specimen leading to impulsive tensile loading due to which the specimen experiences a maximum strain (elongation) and correspondingly, a maximum stress (tension)[7].

A certain procedure is proposed here to determine the impulsive response of this non-linear elastic behavior composite specimen. In this procedure it is necessary first to determine current values of both the strain energy (i.e. energy stored due to elastic deformation) and the Poisson's ratio of the specimen during its incremental elongation calculation in the elastic mode mentioned previously.

3.1. Strain Energy and Effective Poisson's Ratio Determination

Considering applied longitudinal stress in the X-direction, the strain energy (E) is

defined by the integration $E = \int_{0}^{\varepsilon_X} \sigma_X d\varepsilon_X$, this integration is determined numerically using trapezoidal rule after each tensile cycle (step).

Concerning Poisson's ratio, this ratio may be defined effectively for sine-shaped fibers composite from eq.(1) as:

$$\nu_{xy}^{*} = - S_{12}^{*} / S_{11}^{*} \qquad (4)$$

The average compliance (S_{11}^{*}) was formulated in Ref.(4). Making use of some results in that reference, the following expression for (S_{12}^{*}) is concluded here as:

$$S_{12}^{*} = S_{12} + \frac{c/2}{(1+c)^{3/2}}(S_{11} - 2S_{12} + S_{22} - S_{66}) \qquad (5)$$

where, $c=(2\pi a/\lambda)^2$ and S_{ij}s are elastic compliances for straight-fibers condition (i.e. for a given θ, with the tension stress applied parallel to the straight fibers which indicate a principle axes). The coefficients S_{ijs} are related to engineering constants of the composite for straight-fibers condition.

3.2. Proposed Procedure for Impulsive Loading Response Determination

For linear materials (i.e. those of constant Young's moduli), the maximum loading (P) the specimen experiences for the situation shown in Fig.(3) is determined by the known equation[8]:

$$P = W\left[1 + \sqrt{\frac{2hAE_y}{WL}}\right] \qquad (6)$$

where, E_y: a constant Young's modulus, A: specimen initial cross sectional area.

This equation is not suitable for composites which usually are characterized by non-linear stress-strain relations (variable Young's moduli).

For our polymer-based composite of curved fibers, a certain procedure is proposed that makes use of some results obtained from the elastic model mentioned in sec (2).

Among the many parameters provided by that model only four are directly related to impulsive loading determinations which are: strain (ε_x), stress (σ_x), strain energy (E), and the equivalent mass (M) which is the amount of mass associated to a given stress, given as: $M=\sigma_x * A / g$, where g is the acceleration of gravity and A is the cross sectional area of the composite specimen. Here use is made of the effective

Poisson's ratio [eq.(4)] in determining the current values of (A) during each tensile cycle in the elastic model. A complete set of the above four parameters is obtained when composite breaking point [denoted by symbol (b)] is reached, at which the elastic calculations cease and the impulsive calculations may be performed.

To make an impulsive test, given values of falling mass (Mo) and height of fall (h) are provided to the impulsive calculation model in which two parts of energy are determined, namely, static (Es) and impulsive (Ep) energies. The static part is due to the effect of mass-existence only, it is determined by performing an interpolation process to find the strain energy corresponding to the given mass (Mo). This may be illustrated by Fig.(4).

The static energy (Es) is determined as [see Fig.(4)]:

$$E_S = E_i + \frac{E_{i+1} - E_i}{M_{i+1} - M_i}(M_O - M_i) \ .$$

While the impulsive part, which is due to the mass fall through the given height (h) is simply found as: Ep=Mo.g.h . Hence the total amount of energy provided to the specimen in this process is: Eo=Es+Ep.

Maximum stress (σ_{max}) and strain (ε_{max}) correspond to energy (Eo) may be determined by performing the interpolation again making use of the data provided by the elastic model. This is illustrated in Fig.(5).

The maximum stress (for example) is determined as [see Fig.(5)]:

$$\sigma_{max} = \sigma_i + \frac{\sigma_{i+1} - \sigma_i}{E_{i+1} - E_i}(E_O - E_i)$$

A similar relation may be established for the maximum strain.
Different values of mass and height of fall may be chosen and the maximum values of stresses and strains examined by the specimen may be found. When sufficient magnitudes of mass and/or height are given, the total energy (Eo) may exceed the breaking value (Eb) indicating the failure of the specimen.

4. Results and Discussion

The specimen under consideration in the present calculations is a composite of a polymer resin [Polybutylene terephthalate (PBT)] with sine-shaped glass fibers.
The data adopted for PBT and glass are as follows[1]:

	E_L(GPa)	E_T(GPa)	G_{LT}(GPa)	ν_{LT}	ν_{TL}	ε_b%
PBT	2.156		0.77	0.4		50-300
Glass	72.52		29.7	0.22		4

where, E_L: Longitudinal Young's modulus (along fiber's length)
E_T: Transverse Young's modulus (transverse to fiber's length)
G_{LT}: Shear modulus, ν: Poisson's ratio, ε_b: Breaking strain.

Attention will be paid for the results of impulsive loading. Figs. (6-a, and -b) show maximum longitudinal strains and stresses respectively reached by the specimen due to impact of a 30 Kg. mass falling from different heights. Two specimens are considered with different a/λ ratios: 0.2 and 0.1, both have the same volumetric ratio (Vf). Fig.(6-a) shows that the specimen of higher a/λ is more flexible, i.e., exhibits more strains at a given falling height indicating the effect of fibers curvature on composite flexibility. Also it is noted that the composite of higher a/λ fails at higher heights. Fig.(6-b) shows the corresponding stresses induced in those two specimens as functions of height of fall (h). Comparing Figs.(6-a) and (6-b) it is concluded that higher a/λ leads to more stability in impulsive loading, where for higher a/λ we have higher strains and correspondingly lower stresses. However, the composite of higher a/λ may eventually bear more stresses due to it's failure at much higher values of falling heights (h).Hence it is concluded that both flexibility and strength of the composite are proportional to fiber's curvature, i.e. the ratio a/λ.

The effect of the falling mass value is shown in Fig.(7) where two masses are considered: 30 and 25 Kg. impacting a specimen of a/λ=0.2 and Vf=30%. It is shown that the resulted strains are lower for the lower mass as expected, which means naturally that the corresponding stresses behave in a similar manner.

The rule played by fibers volumetric ratio is presented in Figs.(8-a and –b) in which a 30 Kg. mass impacts two specimens of different Vf (Vf=30% and 20%), both have the same a/λ ratio. It is noted from those two figures that higher Vf leads to lower strains and correspondingly higher stresses at a given height of fall. Also it is noted that the composite of higher Vf can bear higher stresses and strains before failure. Hence we get a stiffer composite for a higher volumetric ratio of fibers.

As a conclusion, increasing fibers curvature (a/λ) increases both flexibility and strength of the composite, while increasing the fibers volumetric ratio increases its stiffness.

Unfortunately, published data concerning impulsive loading of curved-fibers composites couldn't be found for comparison purposes. Hence, static loading situation is considered and a comparison is made with results reported in Ref.(2), where tensile strength of a unidirectional epoxy/Kevlar composite of straight Kevlar fibers was measured for different volumetric ratio (Vf) ranging from 26% to 73%.

Volumetric ratio of 40% was considered for example, where the measured tensile strength of the composite specimen was 0.93 GPa taken as an average of nine samples, while the calculated value using rule of mixture (ROM) was 1.22 GPa[2].

The ultimate tensile strain is expected to be the braking strain of the straight Kevlar fibers (~ 3%).

Here, the developed elastic model mentioned in sec.(2) was applied on the experimental case described above. Straight fibers (a/λ=0) where considered first. The model calculations resulted in a maximum tensile stress (σ_x) of 1.8 GPa and a maximum tensile strain (ε_x) of 3.5%. It is believed that this discrepancy in (σ_x) between experimental and calculated values is due to the lack of parametric material data concerning the exact types of epoxy and Kevlar materials employed in Ref.(2). Calculations on sine-shaped fibers composite were performed to demonstrate the influence of fiber curvature on tensile strength and breaking strain of composite specimen.

Tacking a/λ-ratio of (0.1), the maximum calculated tensile stress and strain were respectively (2GPa) and (12%), while for a/λ=0.2, they were respectively (2.3GPa) and (35%).

This comparison shows that composites of curved fibers are capable of bearing higher strains and consequently, higher stresses than straight fibers composites. Accordingly, it is expected that curved fibers composites still have the same superiority in impulsive loading condition compared with straight-fibers composites.

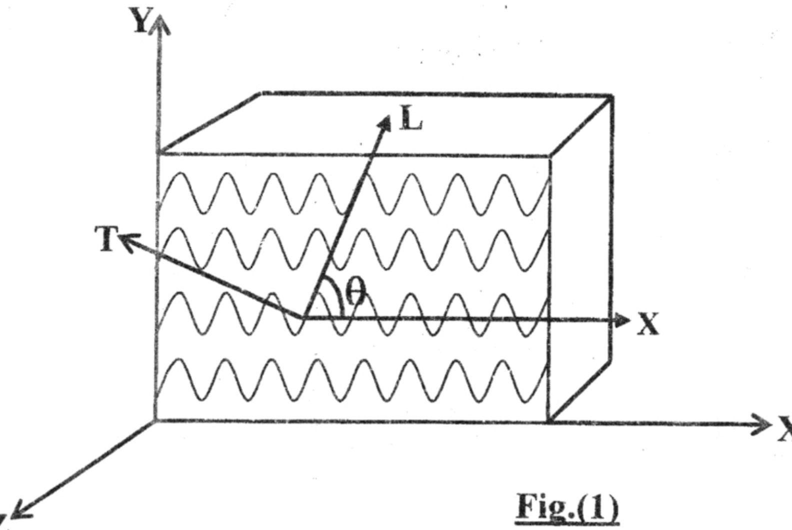

Fig.(1)

Iso-phase configuration of sinusoidal fibers imbedded within a matrix material.

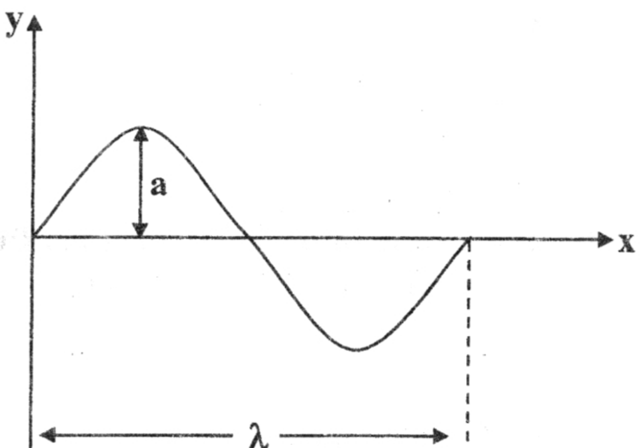

Fig.(2)

A schematic representation of a sine fiber showing its wavelength (λ) and amplitude (a).

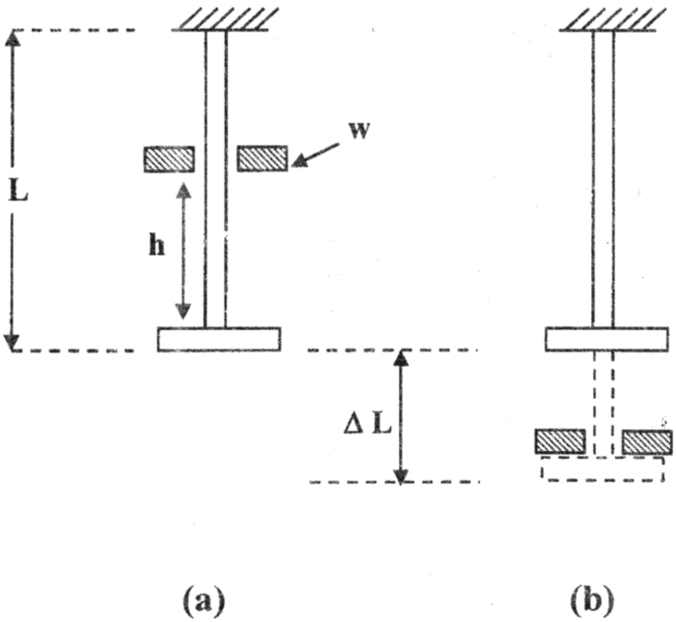

Fig.(3)
A schematic illustration of tensile impact loading
(a): Before impact , (b):After impact.

...	E (J)	M (Kg.)	...
...	E1	M1	...
...	E2	M2	...
...	...	...	...
...	...	...	...
...	Ei	Mi	...
ES (determined) →			← Mo(given)
...	Ei+1	Mi+1	...
...	...	...	...
...	...	...	...
...	Eb	Mb	...

Fig.(4)

A schematic illustration of determining strain energy (Es) that corresponds to a given mass (Mo) via interpolation process on data obtained from elastic model.

...	ε	σ (GPa)	E (J)	...
...	ε_1	σ_1	E1	...
...	ε_2	σ_2	E2	...
...	...	...	...	...
...	...	...	...	...
...	ε_i	σ_i	Ei	...
...	ε_{i+1}	σ_{i+1}	Ei+1	...
...	...	...	...	...
...	...	...	...	...
...	ε_b	σ_b	Eb	...

ε_{max} and σ_{max} (determined)

Eo(given)

Fig. (5)

An illustration of determining σ_{max} and ε_{max} corresponding to the total energy (Eo) via interpolation process.

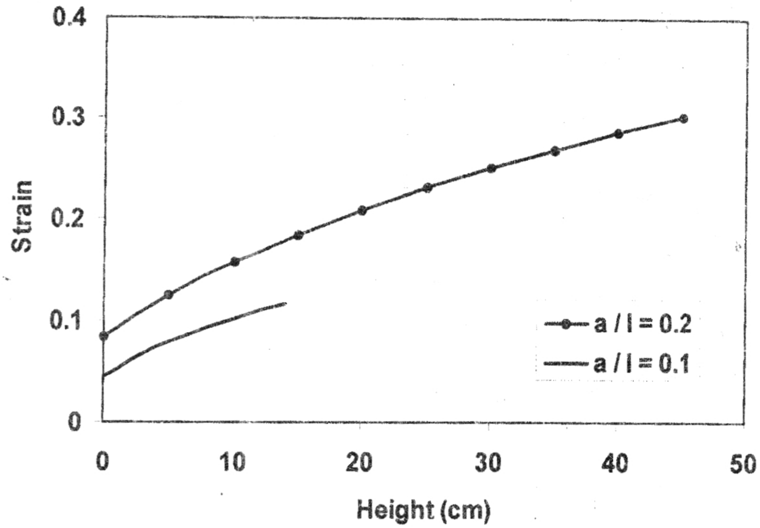

Fig.(6-a)

Tensile strain versus height of fall for two values of a/λ. The falling mass = 30 Kg. and the fibers Vf ratio = 30%.

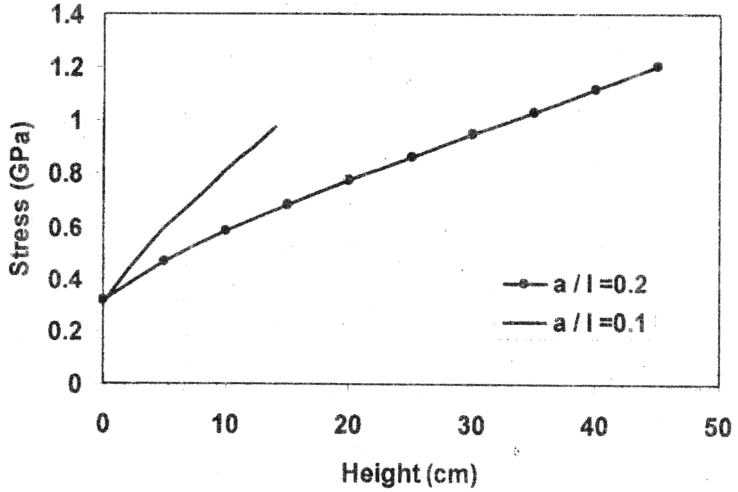

Fig.(6-b)

Tensile stresses corresponding to strains of Fig. (6-a).

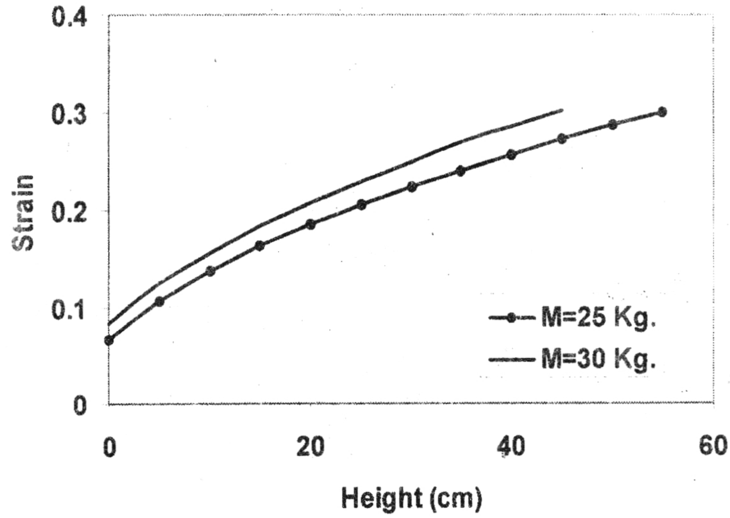

Fig. (7)

Tensile strains versus height of fall for two values of falling mass. The composite specimen is of a/λ= 0.2 and Vf= 30%.

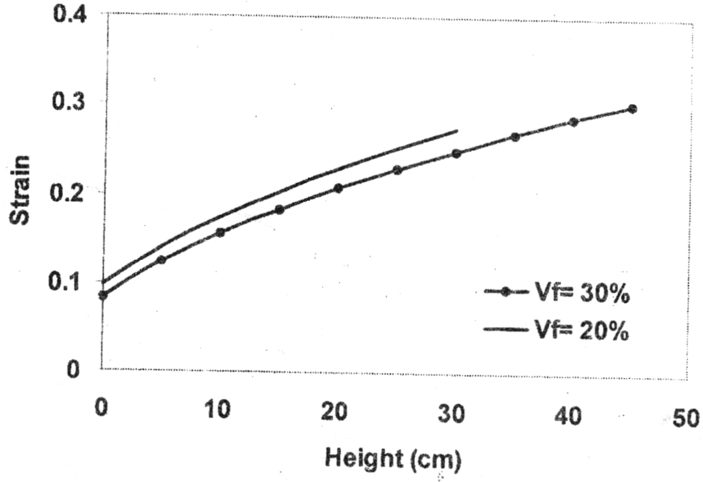

Fig. (8-a)

Tensile strain versus height of fall for two values of Vf. The falling mass = 30 Kg. and a/λ ratio = 0.2.

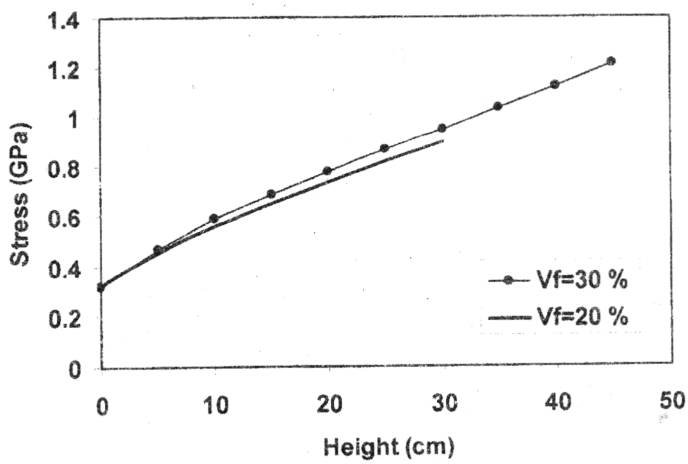

Fig. (8-b)

Tensile stresses corresponding to strains of Fig. (8-a).

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نمذجـــة نظريــــة لتأثيــــر الشـــد النبضي على متراكبات ذات ألياف منحنية

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المستخلص

تم اقتراح أسلوب نظري لأحتساب استجابة متراكبات مرنة بوليمرية ذات ألياف زجاجية متموجة لأجهاد شد نبضي ، حيث جرى اعتماد و تطوير أحد النماذج الخاصة بالنطاق المرن للأجهادات بغية استحصال بعض البيانات الضرورية لأستخذامها في احتساب الإستجابة النبضية في النموذج الحالي.