Spectrum Estimation of Frequency Hopping Signal

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1. Abstract:

This work investigates the estimation of the spectrum, when a frequency hopping (FH-SS) signal had been intercepted and determines the frequency of FH signal at maximum position using finite data records which are of major interest in military and commerical applications. The work has been accomplished using Matlab version 6.5. The FFT algorithm (periodogram) has been used to estimate the frequency of the hops. In the simulation, the frequency spacing between hops is taken to be 1 kHz and two frequencies are tested (50 kHz and 120 kHz).

2. Introduction

Spread spectrum systems, proposed initially for antijam, secure communication, and combating multipath fading, have acquired increased importance as they are applied to mobile communication and other systems through code division multiple access systems. The immediate purpose of frequency estimation is to determine the center frequency and possibly the spectral shape of an intercepted signal. If FH signal is intercepted, the purpose is to determine each hopping frequency or at least the frequency range over which the hopping occurred [1]. The most commonly method used for spectral analysis of FH signal is the Fast Fourier Transform (FFT) (periodogram) because it is easy to perform the calculation and to implement in both software and hardware [2].

3. Frequency Estimation (Spectral Analysis)

The receiver can be modelled as a Fourier transform device, i.e., for input signal x(t), the output will be X(w) as given below [3,4]

$$X(w) = \int_{-\infty}^{\infty} x(t) e^{-(jwt)} dt \qquad \text{..........} \quad \text{......} \quad (1)$$

In fact the input signal x(t) can only be observed for a finite time, thus :

$$X(w) = \int_{-\infty}^{\infty} x(t) w(t) e^{-(jwt)} dt \qquad \text{.........} \qquad (2)$$

where w(t) is the window function.

Equation (2) is the Fourier transform for product of two functions in time domain, which means it represents a convolution in frequency domain. The spectrum given by equation (2) is the desired spectrum given by equation (1) convolved with the Fourier transform of the window function. Thus the spectrum analysis is obtained through the application of FFT algorithm. It is computationaly efficient and produces reasonable results for a large class of signal processing [5].

Most digital computations of spectra have a common procedure as follows [6, 7]:

i- The analog signal is sampled every Ts time unit within a time T thus N = T/Ts digital samples are required.

ii- The samples were multiplied by a window function and Fourier transformed.

iii- The squared magnitude of the resulting transform gives the desired estimation.If {x(n)} is a sampled data sequence which is available for only a finite time window over n = 1,2,3,……., N, then the discrete Fourier transform of x(n) is [8] :

$$DFT\ \{x(n)\} = X(k) = \sum_{n=0}^{N-1} x(n) e^{-(j2pnk/N)} \qquad \text{.......... .....} \qquad (3)$$

From Parsevel theorm [9]

$$\sum_{n=0}^{N-1} \left| x(n) \right|^2 \cdot \Delta t = \sum_{n=0}^{N-1} \left| X(k) \right|^2 \cdot \Delta f \qquad \text{.......... .....} \qquad (4)$$

Also the power spectral density (p.s.d) estimation $S_N(k)$ for a random signal x_N is the discrete Fourier transform (DFT) of the autocorrelation function estimate $R_N(k)$ [10] i.e

$$S_N(k) = \sum_{-\infty}^{\infty} R_N(k) e^{-(j2pnk/N)} \qquad \text{.......... ....} \qquad (5)$$

This definition relates the two estimates, is motivated by the fact that the true PSD and autocorrelation function obey the similer DFT relation:

$$S(k) = \sum_{-\infty}^{\infty} R(k) e^{-(j2pnk/N)} \qquad \text{.......... ....} \qquad (6)$$

The spectral density estimation of (5), can be defined in terms of the sample autocorrelation function, and can be related directly to the observed data. It can be shown that $S_N(k)$ has the following simple relationship to the DFT $X_N(k)$ of the data samples [10]:

$$S_N(k) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x_n e^{-(j2pnk/N)}\right|^2 = \frac{1}{N}\left|X_N(k)\right|^2 \quad .... \quad (7)$$

Equation (7) can be derived by substituting R_N into (5), where:

$$R_N(k) = \frac{1}{N}\sum_{n=-\infty}^{\infty} x_n \cdot x_{n+k} \quad .... \quad .... \quad (8)$$

then $S_N(k)$ become:

$$S_N(k) = \frac{1}{N}\sum_{k=-\infty}^{\infty} \cdot \sum_{n=-\infty}^{\infty} x_n \cdot x_{n+k} e^{-(j2pnk/N)} \quad .... \quad .... \quad (9)$$

When a factor of 1 = e (-j2πnk/N)*e (j2πnk/N) is introduced into (9), the following can be obtained:

$$S_N(k) = \frac{1}{N}\sum_{n=-\infty}^{\infty} x_n e^{-(j2pnk/N)} \cdot \sum_{k=-\infty}^{\infty} x_{n+k} e^{-(j2p(n+k)/N)} \quad .... \quad (10)$$

Changing the summation variable to m = n + k in the second sum, it can be seen that:

$$S_N(k) = \frac{1}{N} X_N(k) \cdot X_N^{*}(k) = \frac{1}{N}\left|X_N(k)\right|^2 \quad .... \quad .... \quad (11)$$

As seen in (7), this spectral density estimate was called a periodogram. The periodogram is seen to be the magnitude squard of the DFT of the data divided by N [10].

Although the FFT algorithm offers a computationally efficient means for numerically evaluating the periodogram, there are several possible fenomenas that result an error between the estimated and exact spectrum. These are [5, 9, 11]:

i- **Aliasing: The high frequency components of a time function can impersonate low frequency components if the sampling rate is too low. The only solution to the alising problem is to ensure that the sampling rate is high enough for the highest frequency component to be sampled at least twice during each cycle.**

ii- **Frequency selectivity: It is the ability to resolve different frequency components of the input signal. It is also called frequency resolution. Components within the bin crossover points of the adjacent bins are not resolvable. The resolution of the FFT is given by**

$$\text{Resolution} = f_s/N \qquad \ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

where

f_s **is the sampling frequency**

and

N here is the length of the data record.

iii- **Spectral Leakage: This problem arises because of the practical requirement that the signal observation should be within a finite interval. A discontinuity resultes at the time boundary of the finite length sequence due to the periodicity property of the FFT. It gives rise to the leakage (spectral contributions). To solve this problem window functions with low sidelobe are selected.**

iv- **Picket-Fence Effect: It also called Scalloping Loss (SL). This effect is produced by the inability of FFT to observe the spectrum as a continous function since computation of the spectrum is limited to integer multiples of the fundamental frequency. If the signal has frequency components between the harmonics of (f_s/N) the amplitudes of these frequency components are reduced. To decrease this problem, the data record length must be increased by adding zeros to give more accurate estimate of the envelope of the FFT, or**

equivalently, a more accurate estimate of the frequency components between the original harmonics at (f_s/N) [11,12].

4. Computer Simulation for Transmitter

The block diagram for FH transmitter is shown in Fig (1), it consists basicaly of two important parts, the PN code generator and the frequency synthesizer.

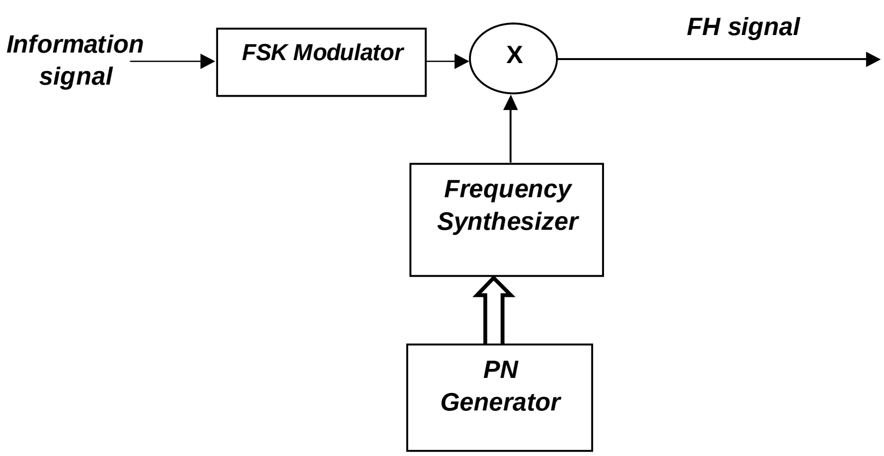

Fig (1) FH transmitter

The pattern of Maximal Length Sequences, or shortly m-sequences, generated by m-stage Linear Feedback Shift Register (ML-LFSR) connection was illustrated in table A.1 in Appendix A. The feedback connection of maximum length sequence is taken at {7, 1} as shown in Fig (2) to generate PN sequence.

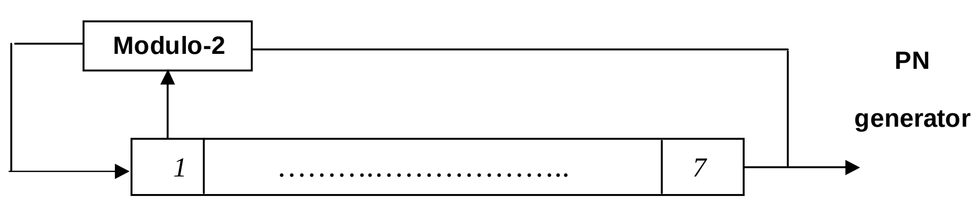

Fig (2) ML-LFSR

The Length of m-sequence is 2^7-1=127 . The frequency synthesizer is shown in Fig (3), (which is practically a PLL system having an M divider in its feedback path) generates frequencies that are integer multiple of reference frequency

corresponding to binary coded decimal (BCD) of the contents of the shift register of PN code generator.

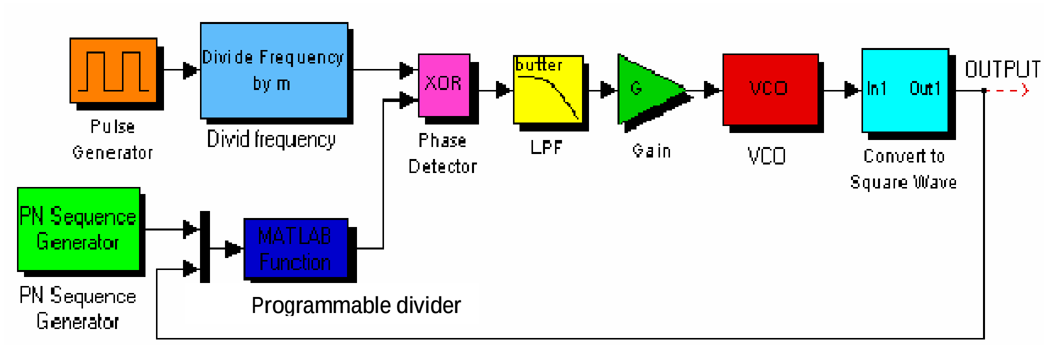

Fig (3) Frequency synthesizer

The generated frequency will hop over a bandwidth of 127 kHz with the channel spacing of 1kHz.

The generated frequency will hop over a bandwidth of 127 kHz with the channel spacing of 1kHz.

The frequency hop signal bandwidth (BW_{ss}) is approximately:

$$M*f_{ref} + BW_{FSK} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

where

$m = 127$

and

$f_{ref} = 1KHz$

The simulation model of transmitter is as shown in Fig (4) where the data stream from Bernoulli binary generators applied to binary FSK modulator with carrier frequencies f_0= 8KHz and f_1= 9KHz.

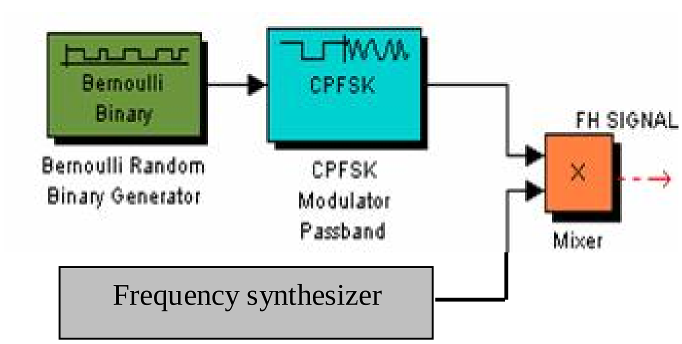

Fig (4) Simulated FH transmitter TRANSMITTER

Here the non spread signal has a bandwidth (BW_{FSK}) of approximately [13,14]

f_1 + rate = 9000 + 10 = 9010 Hz, ………………. (14)

and BW_{ss} =127000+9010 =136010 Hz.

The FSK signal is hopped over different bands by mixing it with a frequency synthesizer output controlled by a PN generator.

5. Computer Simulation of the Frequency Estimation Based on FFT

A computer simulation program to estimate the frequency of the FH signal corresponding to the peak magnitued, based on the FFT method, has been written using Matlab 6.5 Language. The flow chart of the spectral estimation is shown in Fig (5) where the intercepted signal is processed and apply FFT to find the estimated frequency from peak power spectrum. The resolution of the estimator is proportional to the length of the data record as follows:

Resolution = fs/N < 1000 Hz (the frequency spacing between channels)

where

N is the length of the data record and must be power of 2,

f_s = (136010 Hz * 2) = 272020 Hz,

N > fs/resolution > (272020)/(1000),

N > 272.

To be in the safe side, the number of sample points N is taken as $N = 2^9 = 512$ and for good design the error does not exceed the resolution required (1000 Hz). Fig (6) illustrates the spectrum of a noise only, when no signal was sent. Fig (7) illustrates the spectrum of one hop (140 kHz) at the transmitter and after processing at the receiver. Fig (8) to Fig (13) shows the values of the estimated frequency for different SNR, (-15, -10, -5, 0, 5, 10) dB, respectively. The exact frequency value of the hop equal to 50 kHz. The estimated values are acceptable, because the error is about (± 40 Hz), and does not exceed the resolution (1000 Hz). Fig (14) to Fig (19) shows the estimated frequency values for the exact frequency hop (120 kHz) and different SNR. The error does not exceed (± 50 Hz).

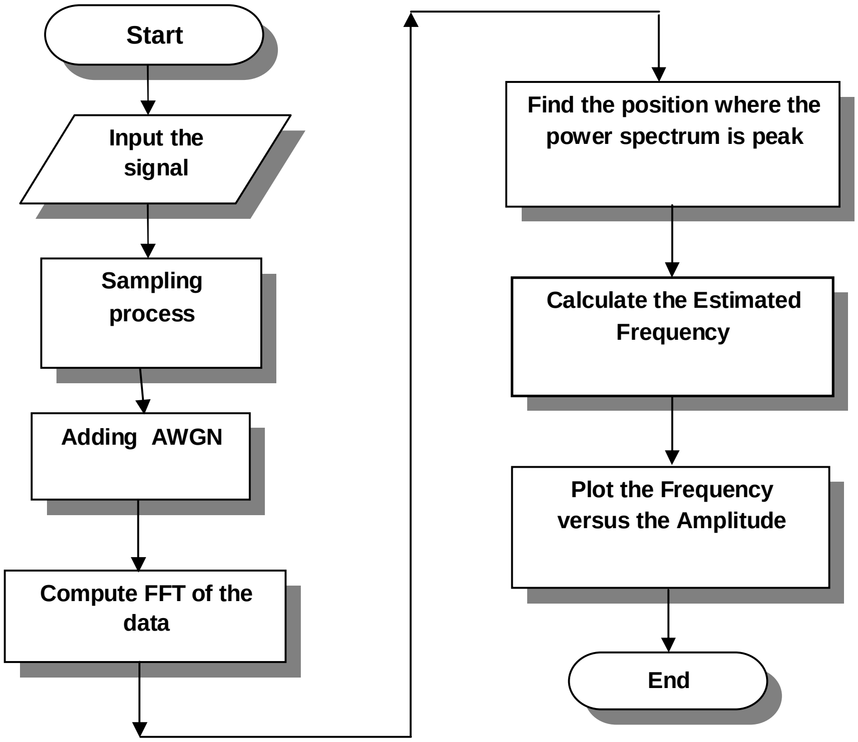

Fig (5) The flow chart for the spectrum estimation

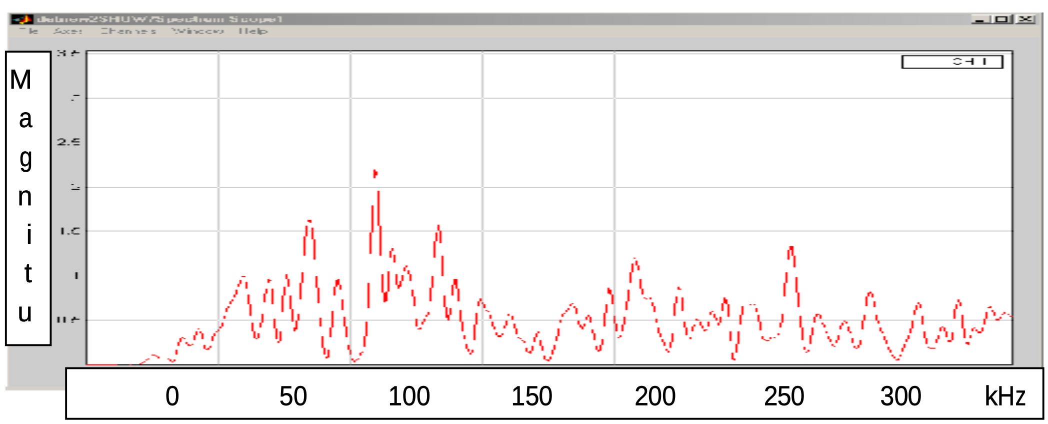

Fig (6) The Spectrum of Noise Only (No signal) at (-8dB)

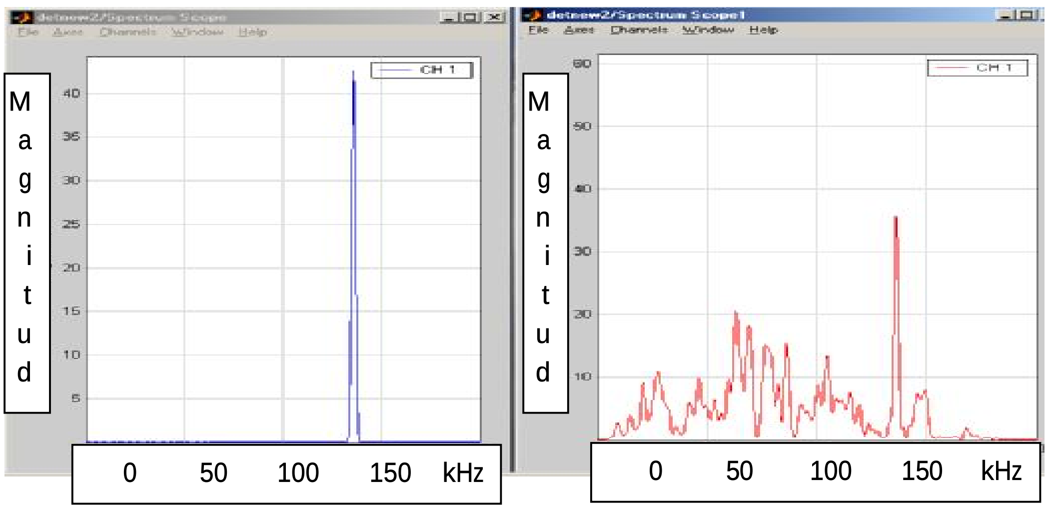

Fig (7 a & b) The Spectrum of One Hop

(a) Input Hopping Signal **(b) Output Noisy Hopping Signal**

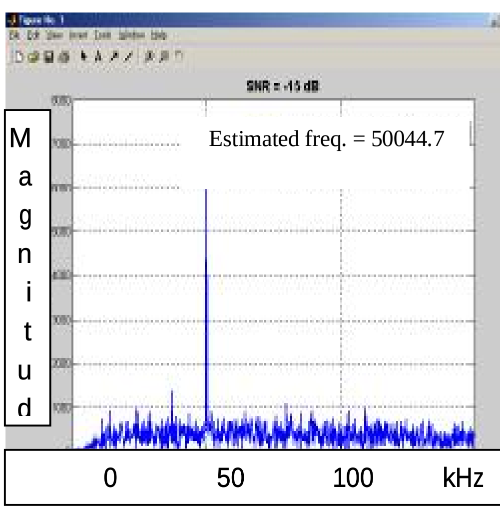

Fig (8) Frequency estimation using FFT at SNR = (-15dB)

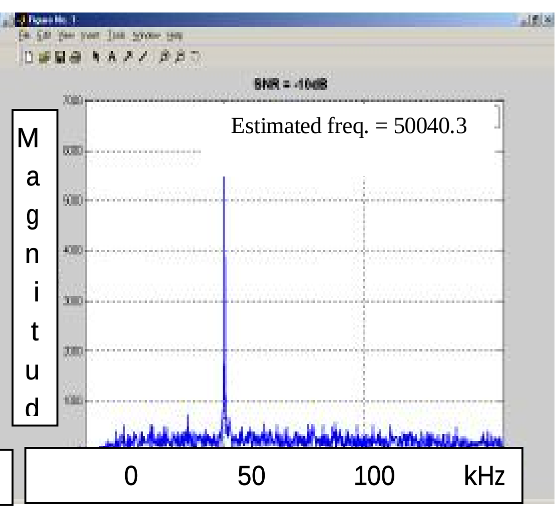

Fig (9) Frequency estimation using FFT at SNR = (-10dB)

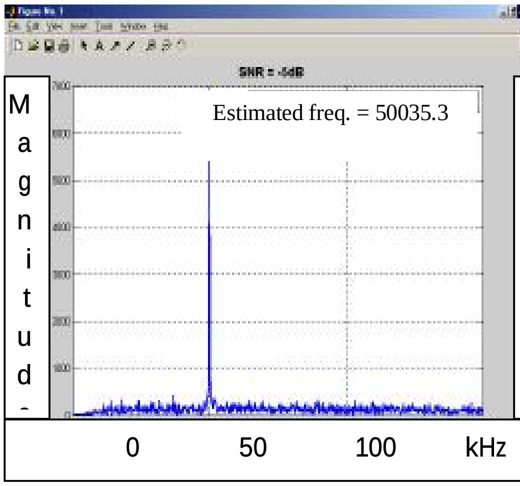

Fig (10) Frequency estimation using FFT at SNR = (-5dB)

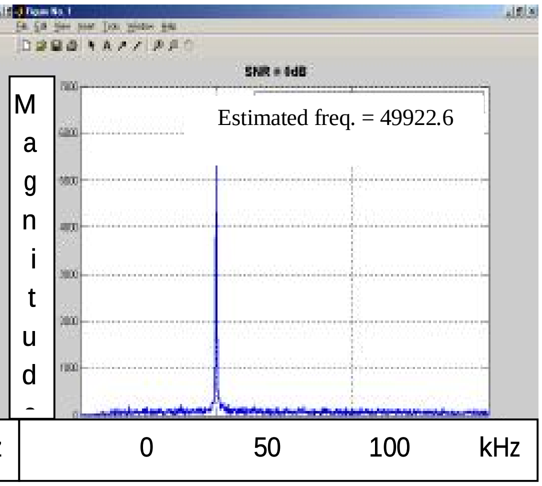

Fig (11) Frequency estimation using FFT at SNR = (0dB)

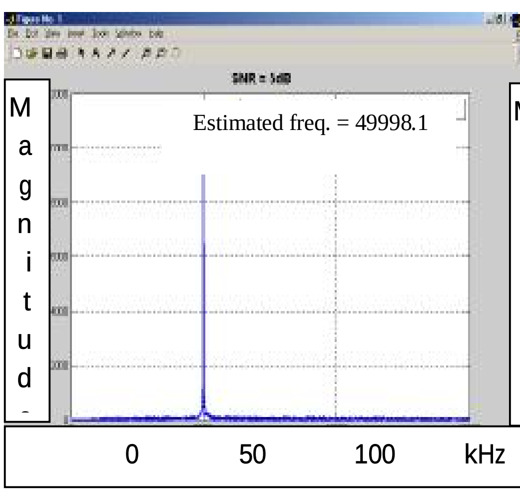

Fig (12) Frequency estimation using FFT at SNR = (5dB)

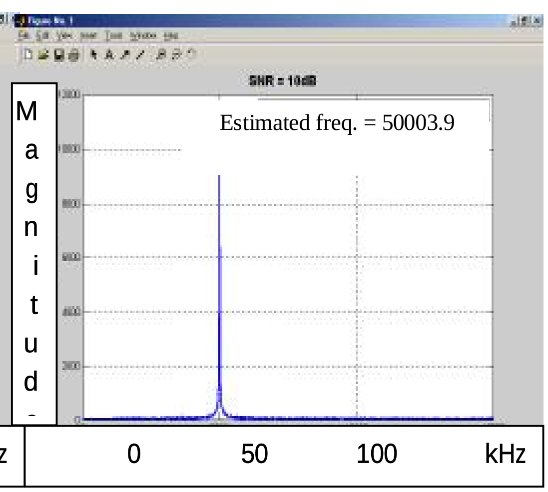

Fig (13) Frequency estimation using FFT at SNR = (10dB)

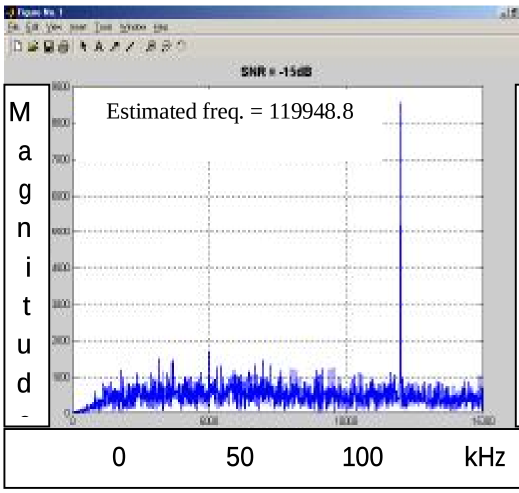

Fig (14) Frequency estimation using FFT at SNR = (-15dB)

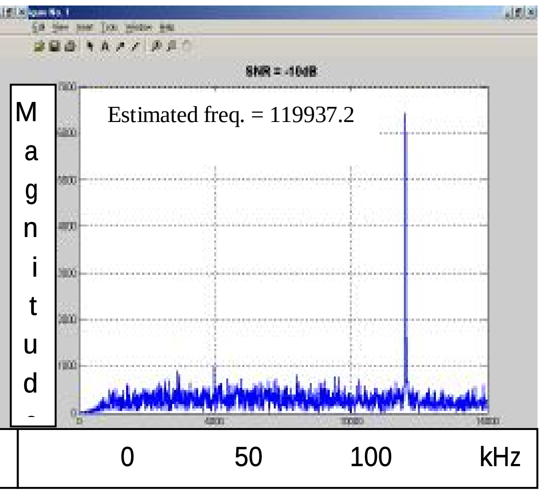

Fig (15) Frequency estimation using FFT at SNR = (-10dB)

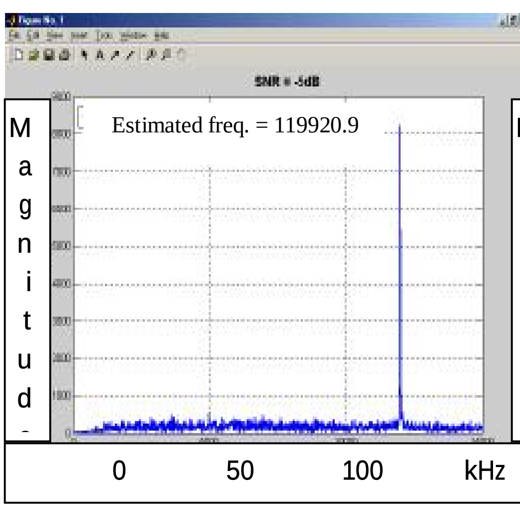

Fig (16) Frequency estimation using FFT at SNR = (-5dB)

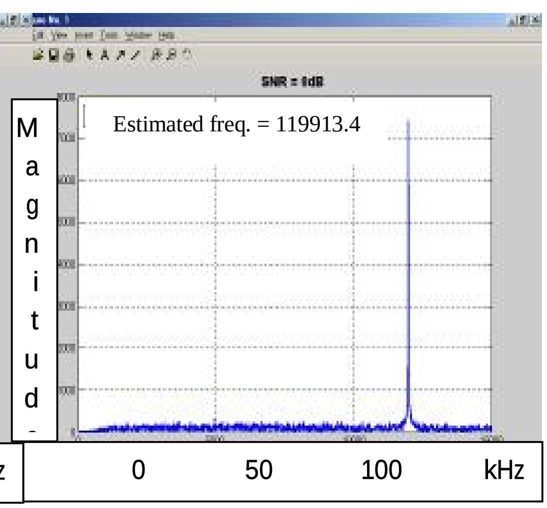

Fig (17) Frequency estimation using FFT at SNR = (0dB)

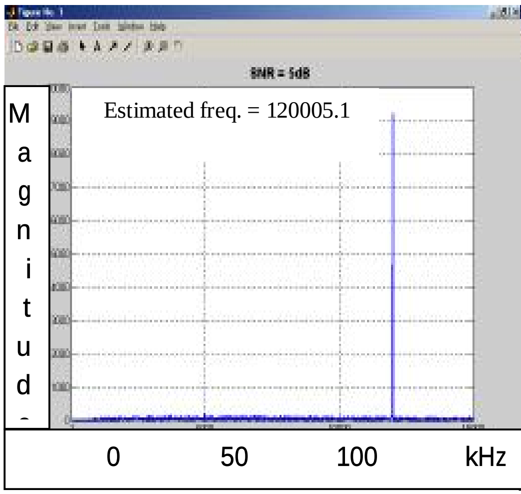

Fig (18) Frequency estimation using FFT at SNR= 5dB

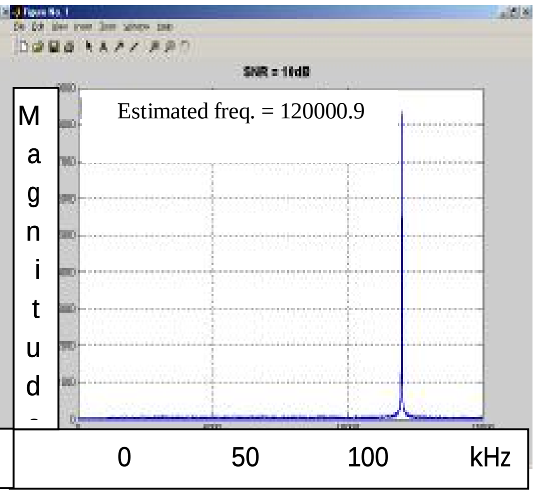

Fig (19) Frequency estimation using FFT at SNR=10dB

6. Conclusions

The spectral analysis or frequency estimation is based on FFT algorithm (periodogram). Two hops from the FH signal has been tested to estimate their frequencies. The first hop has an exact frequency of (50 kHz), and the error in the estimated frequency for different SNR does not exceed (± 40 Hz), and the second hop has an exact frequency of (120 kHz), and the error in the estimated frequency for different SNR does not exceed (± 50 Hz). It is worth to mention that the frequency spacing between two adjacent frequencies is (1 kHz). This result can be considred acceptable, beause the resolution is less than (1 kHz). For the periodogram method, the resolution depends on the data record length which is increased with increasing the sampling points. Also there is leakage problem due to the use a finite data record; but it is simple and must commonly used for spectrum estimation because it gives good estimation for low SNR. The number of multiplications is N log2(N) where N is the number of sampling points.

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Appendix A

Feedback Connection for Liner m-Sequence

A.1 Maximal Length PN Sequences

Maximal Length Sequences, or shortly m-sequences, generated by m-stage Linear Feedback Shift Register (LFSR) are periodic with period length $m = 2^n - 1$ of all possible register states excluding the all zero state .

Table (A.1) Feedback Connection for Linear m-Sequences.

Number of Stages (n)	Code Length (m)	Maximal Tape
2	3	[2,1]
3	7	[3,1]
4	15	[4,1]
5	31	[5,2],[5,4,3,2],[5,4,2,1]
6	63	[6,1],[6,5,2,1],[6,5,3,2]
7	127	[7,1],[7,3],[7,3,2,1],[7,4,3,2], [7,6,4,2],[7,6,3,1],[7,6,5,2]
8	255	[8,4,,3,2],[8,6,5,3],[8,6,5,2] [8,5,3,1],[8,6,5,1],[8,6,5,2]
9	511	[9,4],[9,6,4,3],[9,8,5,4],[9,8,4,1] [9,8,7,2], [9,5,3,2], [9,8,6,5],[9,6,5,4,2]
10	1023	[10,3],[10,8,3,2],[10,4,3,1], [10,8,5,1][10,5,2,1], [10,8,5,4],[10,9,4,2],[10,9,4,1],[10,8,4,3],[10,5,3,2]
11	2047	[11,1],[11,8,5,2],[11,7,3,2], [11,6,5,1] [11,5,3,5], [11,10,3,2], [11,5,3,1], [11,9,4,1], [11,8,6,2], [11,9,8,3]
12	4095	[12,6,4,1], [12,9,3,2], [12,11,10,5,2,1], [12,11,9,7,6,5], [12,11,9,5,3,1], [12,11,9,8,74], [12,11,9,7,6,5], [12,9,8,3,2,1], [12,10,9,8,6,3]
13	8191	[13,4,3,1], [13,10,9,7,5,4], [13,9,8,7,5,1], [13,12,6,5,4,3], [13,11,8,7,4,1], [13,12,6,5,4,3] [13,12,11,9,5,3], [13,12,11,5,2,1], [13,12,9,8,4,2], [13,8,7,4,3,2]
14	16383	[14,12,2,1], [14,13,4,2], [14,11,6,1] [14,12,11,1], [14,6,4,2], [14,11,9,6,5,2]

تخمين الطيف لاشارة القفز الترددي

م.م. عمار عبد الحميد خضر

جامعة الموصل

المستخلص :

إن الطيف المنتشر اقترح اساسا ليكون مضادا للتشويش ومقاوما للانتشار متعدد المسارات ومفيدا لسرية الاتصالات. كما ازدادت أهميته عندما تم تطبيقه في الاتصالات المحمولة وانظمة اخرى من خلال التشفير باستخدام مبدأ تعدد الوصول بتقسيم التردد CDMA .

إن هذا البحث يتناول دراسة تخمين طيف الاشارة بعد ان يتم مقاطعتها وتصنيفها كإشارة قفز ترددي وذلك من خلال طريقة تحويل فورير السريع (FFT) حيث تحتسب قيمة اعلى قدرة لعدد محدد من قيم التسجيل المحللة وتظهر في الاخراج على أنها قيمة الطيف المخمنة. إن معرفة قيمة طيف الاشارة ذات فائدة كبيرة في التطبيقات التجارية والعسكرية.

إن هذا البحث انجز باستخدام تقنية محاكاة المنظومات بالحاسبة.Matlab Simulink

إن القيم التي تم فحصها كانت لاشارتي قفز ترددي 50 kHz & 120 kHz وان الفاصل بين القفزات كان بمقدار kHz ولقيم مختلفة من الضوضاء الكاوزية المضافة في قناة النقل وكانت قيمة الخطأ في التخمين قليلة جدا" ولقد تم تصميم منظومة الإرسال والاستقبال ومركب الترددات لكي يتم توليد اشارة القفز الترددي ومن ثم اعتراضها وتحليلها.