Formation Energy of Alpha-Cluster Pre-formation Probability for Cluster-Formation Model

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Abstract: The pre-formation factor, pre-formation probability or the amount of alpha clustering was recently and well determined using the cluster-formation model (CFM) for alpha decay nuclei. Alpha-cluster formation energy was proposed and used in this model to calculate the factor that is important to calculate the alpha decay width. A new formula for this energy was derived and its validity is investigated. This formula was successful to describe alpha clustering and it can be included in the calculation of any future work.

Keywords: Cluster- formation model, alpha cluster formation energy, alpha decay nuclei, pre-formation factor

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1. Introduction

Alpha-cluster formation energy was first proposed for the cluster-formation model (CFM) that was first presented by Saad et al (2013) [1, 2] to determine the alpha cluster pre-formation probability for even-even heavy nuclei. The pre-formation probability (or pre-formation factor) is a significant factor used in the calculation of alpha decay width. It is defined as the total energy responsible for binding the four nucleons to form alpha particle when alpha particle is inside the alpha decay nuclei. The pre-formation factor was evaluated and chosen constant for even-even nuclei for many decades however many researchers ascertain that this factor has rich information about the nuclear structure because it maps the clustering inside the nuclei before the decay. Many works adapted this model to determine the pre-formation probability for even-even, odd-A, and odd-odd heavy and super heavy nuclei [3-7].

In cluster-formation model, the pre-formation probability P_a is a ratio of the formation energy of alpha cluster E_a to the alpha decay energy (the nucleon surface energy) E_s.

$$P_a = \frac{E_a}{E_s} \tag{1}$$

The privilege of this method is the determination of these two energies from the binding energy of nuclei obtained from the mass defect. The formation energy of alpha was first calculated as a difference between the alpha decay energy and the Q-value of alpha decay for determining the P_a for ^{212}Po. It was expressed as

$$E_a(A,Z) = 2B(A,Z) - 2B(A-4,Z-2) - B(4,2) \tag{2}$$

Where $B(A,Z)$ is the binding energy of a nucleus of mass number A and atomic mas Z. The calculation yielded acceptable value of P_a which was very close to that obtained from experimental alpha decay width and a calculated penetration probability using Wentzel-Kramers-Brillouin (WKB) method [1]. But, this determination of E_a was not successful to describe P_a for a group of even-even heavy nuclei [2]. When the nucleon-nucleon interaction energy formulae (E_{n-n}, E_{p-p} *and* E_{p-n}) from the binding energy was used in the derivation of the formation energy, the alpha clustering for a group of even-even nuclei was well described [2]. These formulae were evaluated from also the binding energy as

$$E_{p-p} = B(A,Z) + B(A-2,Z-2) - 2B(A-1,Z-1) \quad (3)$$

$$E_{n-n} = B(A,Z) + B(A-2,Z) - 2B(A-1,Z) \quad (4)$$

$$E_{p-n} = B(A,Z) + B(A-2,Z-1) - B(A-1,Z-1) - B(A-1,Z) \quad (5)$$

$$E_{\alpha} = 3B(A,Z) + B(A-4,Z-2) - 2B(A-1,Z-1) - 2B(A-1,Z) \quad (6)$$

The last equation was also used to determine the pre-formation probability of alpha cluster for even-even super-heavy nuclei. The results well described the nuclei and confirmed the similarity of alpha formation between the heavy and super-heavy nuclei [6,8,9]. The extension work to the determination of P_a for odd-A and odd-odd nuclei required an investigation of this alpha-formation energy because of the different number of nucleons in the nuclear surface [4,5,10,11]. Deng et al. rewrote the formula of alpha formation energy for even-even nuclei (Eq. 6) in terms of the nucleon separation energy to rewrite it again according to the symbolic symmetry for odd-A and odd-odd nuclei. However this determination was based on no physics, it violates the basics of the cluster-formation model surface [4,10].

Although this violation was reported in 2017 when I presented the correct formulae for the nuclei with details of derivation [5], Ge et al. unfortunately used this incorrect formula [11]. The derivation revealed that the formation energy of odd-A and odd-odd nuclei have the same value of even-even nuclei. This conflict pushed me to investigate the formation energy again.

2. Derivation of the alpha-formation energy

For a nucleus of a mass and an atomic numbers A and and Z, the formation energy E_a of alpha cluster from Eq. (6) was derived starting from the definition

$$E_{\alpha} = E_{n1-n2} + E_{n1-p1} + E_{n1-p2} + E_{n2-p1} + E_{n2-p2} + E_{p1-p2} \quad (7)$$

Where $E_{x\text{-}y}$ is the interaction energy between the nucleons x and y. After substituting Eqs. (3,4 and 5) in Eq. (7), Eq. (6) is obtain as shown in Ref. [2,5].

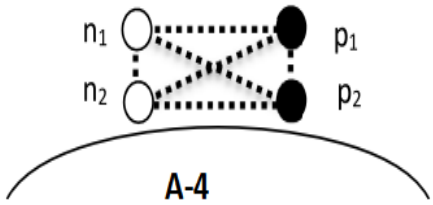

Figure 1: The parent alpha decay nucleus with the last four nucleons that form alpha particle before emission.

The investigation is to derive this equation starting from writing the total binding energy of this system as a sum of the energy in (A-4),the interaction of each nucleon x with the A-4 nucleus ($E_{x\text{-}(A\text{-}4)}$) and the interactions between each two nucleons

$$B(A,Z) = B(A-4,Z-2) + E_{p1-(A-4)} + E_{p2-(A-4)} + E_{n1-(A-4)} + E_{p2-(A-4)} + E_{n1-n2} + E_{n1-p1} + E_{n1-p2} + E_{n2-p1} + E_{n2-p2} + E_{p1-p2} \quad (8)$$

When the last six terms is substituted by the formation energy E_a as in Eq. (7) and solving for it, we obtain

$$E_a(A,Z) = B(A,Z) - B(A-4,Z-2) - E_{p1-(A-4)} - E_{p2-(A-4)} - E_{n1-(A-4)} - E_{p2-(A-4)} \quad (9)$$

The energy $E_{x\text{-}(A\text{-}4)}$ is equivalent to the separation energy of the last nucleon: S_p and S_n, which can be written as

$$E_{p(1,2)-(A-4)} = S_p = B(A-3,Z-1) - B(A-4,Z-2) \quad (10)$$

$$E_{n(1,2)-(A-4)} = S_n = B(A-3,Z-2) - B(A-4,Z-2) \quad (11)$$

Substituting these last two equation in Eq. (9), I obtain

$$E_a(A,Z) = B(A,Z) + 3B(A-4,Z-2) - 2B(A-3,Z-1) - 2B(A-3,Z-2) \quad (12)$$

This equation obtained is to calculate the alpha cluster formation energy. Comparing this equation with Eq. (6), the formula (Eq. (12)) is supposed to reproduce the binding energy of the free alpha when A=4 and Z = 2 to obtain $E_a(4,2) = B(4,2)$.

3. Discussion

To compare the new formula (Eq. (12)) with the Eq. (6) to investigate the validity, the difference is supposed to be determined. When I made some derivation to find the difference in terms of binding energy, it leads to no sense. It is important to keep the form of this formula unchanged because most of its terms are for nuclei close to the daughter nuclei. So, a comparison can be achieved when the energy and the pre-formation factor of some nuclei are calculated. In Table 1, three nuclei were chosen to calculate the alpha formation energy and the pre-formation factor. The typical nuclei of double closure ^{212}Po, the N-drip line ^{218}Po, Z-drip line ^{212}Th and N-drip line ^{236}Th nuclei can show the variance in the calculated energy and the pre-formation probability. As can be seen in Table 1, the difference between the energy from Eq. (6) and Eq. (12) obtain in this work is slight to lead to a percentage difference at maximum 10%. The maximum difference is in the typical nucleus that had a theoretical value pre-formation factor of 0.23-0.30 [12–14], however the formula obtained in this work is still good to reproduce the value. Other nuclei with different properties of binding energy have less percentage difference, which indicates the validity of the Eq. (12). This formula also can be used to determine the alpha clustering formation required to obtain accurate values of pre-formation factor that is used to calculate the alpha decay life times of nuclei. So, the two formulae can both be used to calculate the energy by taking the average. This calculation is similar to the calculation of pairing energy from upper and lower nuclei to obtain more accurate values.

Table 1: Results of alpha cluster formation energy E_a and the pre-formation probability P_a using the two formulae in Eq. (6) and Eq. (12).

Nuclei	Properties	E_a (keV)		P_a		Percentage difference of P_a
		Eq. (6)	Eq. (12)	using Formula of Eq. (6)	using Formula of Eq. (12)	
^{212}Po	typical nuclei	4275	3869	0.221	0.200	10%
^{218}Po	Z=84, N-drip line	4735	4484	0.214	0.202	6%
^{212}Th	Z=90, Z-drip line	4234	4165	0.208	0.204	2%
^{236}Th	N-drip line	3750	3892	0.150	0.156	4%

4. Conclusions

The alpha-cluster formation energy for the cluster-formation model is derived in a different way to obtain a new formula. This formula well describes the clustering in alpha particle inside the parent alpha decay nuclei. The new formula and the previous one can both be used and obtain the average value to obtain more accurate results. The significance of the new formula (Eq. (12)) is to determine the energy from nuclei close to the daughter whereas in Eq. (6) from nuclei close to the parent nuclei. This is useful when the data of the parent is unavailable or inaccurate as in super-heavy nuclei, which helps to decrease the inaccuracy in results. This formula with the previous one provide more accurate pre-formation factor required for the calculation of alpha decay width especially for the super-heavy nuclei.

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طاقة تكوين احتمالية انشاء عنقود الفا لنموذج تكون العنقود

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المستخلص: يبدو أن تطوير البرمجيات آخذ في الازدياد ، بسبب الحاجة المتزايدة لأن تدخل الأعمال التجارية عبر الإنترنت. هناك دفعة لتطوير التطبيقات التي تمت ترقيتها من شأنها أن تساعد الشركات على أن تصبح أكثر كفاءة ونموًا. تشتمل هياكل البرمجيات الحالية على خدمات متجانسة وصغيرة ، والتي تحظى بشعبية وقوية. بالنسبة لبنية الخدمات المصغرة ، فإنها توفر مزايا عملية مثل قابلية التوسع والمرونة ، فضلاً عن كونها وسيلة فعالة من حيث التكلفة لتطوير التطبيقات الكبيرة. على الجانب الآخر ، يفقد النهج المتآلف تفضيله لأنه يعرض منهجيات توصيل البرامج الحالية للخطر. في هذه الورقة ، سنناقش الاختلافات بين الخدمات المصغرة والبنى المتجانسة ، مع إبراز نقاط القوة والضعف في كل منهما ، والتفكير في المقارنة اعتمادًا على اختيار هيكل بسيط لتطبيق السفر ، والذي يفوز باختياره كأفضل خيار في عالم البرمجيات التجارية.

الكلمات المفتاحية: هندسة الخدمات المصغرة، الهندسة المعمارية المتجانسة، هندسة البرمجيات، التطبيق

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