The Nature of the Solution for Cancerous Tumors Operator Equation

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Abstract: In this paper, we study the solution of the cancerous tumors operator equation of the form: $A^* XA + t\, AXA = Q$, Where A is a known operator defined on a Hilbert space H, represent the contributing factors are cancer (Hormonal factor ,Nutrient factor, Relationship between nutrient and hormone, Genetic factor, Viruses,...), Q is a known operator defined on a Hilbert space H, represent the percentages of contributing. Factors are cancer (Sex, Age, Exposure to radiation, Take sedatives, Psychological stress, Factors that weaken immunity, ...), t is any scalar positive real number, represent the control factor, X is unknown operator, represents the treatment of the injured in terms of quantity and quality of treatment , and A^* is adjoint of operator A. Also, we show the nature of the solution for this operator equation for special types of operators. As well as, we will introduce some propositions and remarks because it is very important in this new study of medical systems of differential equations.

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1. Introduction

In general , the operator equations play an important rule in control theory (medical systems of differential equations) , [1 - 2].

These types of operator equations have many real life applications in physics , weather , atmospheric models, and medical systems , [1 - 2]

The operator equation of the form:

$$A^* XA + t\, AXA = Q \tag{1.1}$$

is one of the generalization of the continuous – time Lyapunou equation (the cancerous tumors operator equation)

Where A, Q are given operators defined on a Hilbert space H ,t is any scalar and X is the unknown operator .

The authors in [1],[4] and [5] studied the necessary and sufficient conditions for the solvability of the operator equations of the form :

$$AX + XB = W \tag{1.2}$$

$$AX + XA = W \tag{1.3}$$

and

$$AXB + BXA = W \tag{1.4}$$

where A,B and W are known operators on a Hilbert space H and X is the unknown operator that must be determined.

In this paper, the nature of the solution of the above equation is studied for special types of operators. Also, we study and discuss the range of τ_{A} where :

$$\tau_{\mathrm{A}}(X) = A^* XA + t\, AXA$$

And we prove that τ_{A} neither derivation and nor $* -derivation$.

2. The Nature of the Solution for Cancerous Tumors Operator Equation

In this section , we study the nature of the solution of eq.(1.1) for special types of operators.

Proposition (2.1)

If A,Q are self – adjoint operators and t is any scalar (positive real number) and eq. (1.1) has only one solution then this solution is also self – adjoint .

Proof

Consider eq.(1.1) since $A^* = A$ and $Q^* = Q$ then

$$A^* XA + t\, AXA = Q$$

$$(A^* XA + t\, AXA)^* = Q^*$$

$$A^* X^*A + t\, A^*X^*A^* = Q^*$$

$$A^* X^*A + tAX^*A = Q^*$$

And since eq.(1.1) has only one solution this $X^* = X$, so X is self – adjoint

Remark (2.2)_ if Q is self – adjoint operator , A is any operator and t is any scalar , then the solution of eq.(1.1) is not necessarily self- adjoint operator .

Remark (2.3) if A is self – adjoint operator , Q is any operator and t is any scalar , then the solution od eq.(1.1) is not necessarily self – adjoint operator.

Propostion (2.4) if A ,Q are skew –adjoint operators , t is any scalar (positive real number)and eq.(1.1) has only one solution , then this solution is skew- adjoint.

Proof

Consider eq.(1.1) since $A^* = -A$ and $Q^* = -Q$

then

$$A^* XA + t\, AXA = Q$$

$$-(A^* XA + t\,AXA)^* = -Q^*$$

$$-A^* X^*A - t\,A^*X^*A^* = -Q^*$$

$$A^*(-X^*)A + t\,A^*(-X^*)A^* = -Q^*$$

$$A^*(-X^*)A + t\,(-A)(-X^*)(-A) = Q$$

$$A^*(-X^*)A + t\,A(-X^*)A = Q$$

and since eq.(1.1) has only one solution this $-X^* = X$ so X is skew – adjoint .

Proposition (2.5)
Consider eq.(1.1) , if A is a compact operator , then this equation is compact.

Proof
Consider eq.(1.1) , since A is compact operator , then A^* is compact , so $A^* XA$is compact . Also , A is compact , then AX is compact and $t\,A\,XA$ is compact , since $A^* XA$ and $t\,A\,XA$ are compact , then $(A^* XA + t\,A\,XA)$ compact , so Q is compact .

Remarks (2.6)
1- If A,Q are compact operators and t is any scalar , then the solution X of eq.(1.1) is not necessarily compact operator.

2- If Q is a compact operator , and t is any scalar , then A and X in eq.(1.1) are not necessarily compact operators.

3- If A or Q compact operators , t is any scalar and the solution X of eq.(1.1) exists , then the solution X is not necessarily to be compact.

4- If A and Q are compact operator , t is any scalar and the solution X of eq.(1.1) exists, then the solution X is not necessary to be compact .

Remarks (2.7)
1- If A and Q are normal operators , t is any scalar , then the solution X of eq.(1.1) is not necessarily exists.

2- If Q is a normal operator , A is any operator , and t is any scalar , then it is not necessarily that the solution X of eq.(1.1) is normal operator.

3- If A and Q are normal operators , and t is any scalar , if there exists of eq.(1.1) is not necessarily that the solution X is normal operator.

3. Special Cases for The Solution of Cancerous Tumors Operator Equation.

In this section, we introduce special cases for the solution of cancerous tumors equation. Recall that the spectrum of operator $\equiv \ \sigma\,(A)$.

$\sigma\,(A) = \{\ \lambda \epsilon ¢ : A - \ \lambda I$ is not invertible }

And B(H) is the Banah space of all bounded linear operators defined on a Hilbert Space, [3].

Theorem (3.1) , [1] : If A and B are operators in B(H), such that $\sigma(A) \cap \sigma(B) = \emptyset$, then the operator equation $AX - XB = C$ has a unique solution X, for every operator C

The following propositions give the unique solution for the operator eq.(1.1) .

proposition (3.3) If A is an identity operator , and Q is any operator in B(H) , then the solution of eq.(1.1) is $X \ = \frac{1}{(1+t)}\ Q$.

Proof
consider eq.(1.1) , and $A = I$ (identity operator)

$$A^*\,XA + t\ AXA = Q$$

$$I^*\,XI + t\ IXI = Q$$

$$X\ + t\,X\ = Q$$

$$(1 + t\,)X\ = Q$$

Then
$$X\ = \frac{1}{(t+1)}\ Q$$

Corollary (3.2) If A is an invertible operator from right in B(H) ,

$\sigma(A) \cap \sigma(-tA) = \emptyset$, Q is any operator in B(H) and t is any scalar, then the operator eq.(1.1) has a unique solution.

Proposition (3.4) If A is an invertible self- adjoint operator in B(H), Q is any operator in B(H) and t is any scalar, then the solution of eq.(1.1) is $X = \frac{1}{(1+t)} A^{-1} Q A^{-1}$

Proof

Consider the operator eq.(1.1)

$$A^* XA + t\, AXA = Q$$

$$A^{-1}A^* XA + tA^{-1} AXA = A^{-1}Q$$

$$I\, XA + tI\, XA = A^{-1}Q$$

$$XA + tXA = A^{-1}Q$$

$$X\, A\, A^{-1} + tX\, A\, A^{-1} = A^{-1}Q\, A^{-1}$$

$$XI + tX\, I = A^{-1}Q\, A^{-1}$$

$$(X + tX) = A^{-1}Q\, A^{-1}$$

Then $X = \frac{1}{(1+t)} A^{-1} Q A^{-1}$

4. On The Range of τ_A

Recall that,a linear mapping τ from a ring R to itself is called a derivation, if $\tau_{ab} = a\, \tau_b + \tau_a\, b$, for all a, b in R ,[2].

Now, we study and discuss the range of τ_A , where $\tau_A = A^* XA + t\, AXA$.

And, we prove that the range τ_A is neither derivation and nor * -derivation .

Now, define the mapping

$\tau : B(H) \to B(H)$ by :

$$\tau(X) = \tau_A(X) = A^* XA + t\, AXA$$

$X \epsilon B(H)$, where A is a fixed operator in B(H) , and t is any scalar.

It is clear that the map $\tau_A(X)$ is a linear map also , the map τ_A is bounded, in fact

$$\| \tau_A \| = \| A^* XA + t\, AXA \quad \| \leq \| A^* XA \| + |t| \; \| AXA \|$$

$\leq \| A^* \| \| X \| \| A \| + t \| A \| \| X \| \| A \| \leq \| A \|^2 \| X \| + t \; \| A \|^2 \| X \| \leq (1+t) \; \| A \|^2 \| X \|$

Since $\| A^* \| = \| A \|$

Put $M = (1+t) \; \| A \|^2$, which is non negative number.

So $\| A^* XA + t\, AXA \; \| \leq M \; \| X \|$, then τ_A is bounded.

The following steps shows that :

$[Range\ (\tau_A)]^* \neq Range\ (\tau_A)$.

$$[Range\ (\tau_A)]^* = \{(A^* XA + t\, AXA)^* : X \epsilon B(H)\}$$

$$= \{A^* X^* A + t\, A^* X^* A^* : X \epsilon B(H) \}$$

Let $X_1 = X^*$

$$= \{A^* X_1 A + t\, A^* X_1 \; A^* : X_1 \epsilon B(H) \}$$

$$\neq Range\ (\tau_A)$$

Also

$$\alpha\, Range\ (\tau_A) = \{\alpha\, A^* XA + \alpha t\, AXA \; , X \epsilon B(H) \} \quad ,$$

$$= \{A^* (\alpha X) A + t\, A\, (\alpha X) A \, , X \epsilon B(H)\} \quad .$$

For every α is any scalar,

Let $X_1 = \alpha\, X$

$$= \{A^* X_1 A + t\, A^* X_1 \; A^* \, , X_1 \epsilon B(H)\} ,$$

$$\alpha\, Range\ (\tau_A) = Range\ (\tau_A) \quad .$$

The following remark shows the mapping τ_A is not a derivation :

Remark (4.1) since

$$\tau_A(XY) = A^*(XY)A + t\,A\,(XY)A \;,$$

For all $X, Y \epsilon B(H)$ and

$$X\,\tau_A(Y) = X\,A^*\,Y\,A + t\,XAYA \quad ,$$

also

$$\tau_A(X)Y = A^*\,X\,AY + tAXAY,$$

then one can dedue that

$$\tau_A(XY) \neq X\,\tau_A\,(Y) + \tau_A(X)Y\,.$$

Now , the following remark shows the mapping τ_A is not $* -derivation$:

Rcall that , a Jordan $f: R \to R$ is defined to be an additive mapping satisfy $f\,(a^2) = af(a) + f(a)a$. NOW ,Let R be $* -ring$,a ring with involution $*$. A linear mapping $\tau : R \to R$ is called Jordan $* -derivation$, if for all $a, b \in R$ and $\tau\,(a^2) = a\tau(a) + \tau(a)a^*$, [2].

Remark (4.2)
Since

$$\tau_A\,(X+Y) = A^*\,(X+Y)A + t\,A\,(X+Y)A$$

$$= A^*\,X\,A + A^*\,Y\,A + t\,A\,X\,A + t\,AYA$$

$$= \tau_A\,(X) + \tau_A(Y).$$

Now,

$$X\,\tau_A\,(X) + \tau_A(X)\,X^* = X\,A^*\,X\,A + X\,t\,A\,X\,A + A^*\,X\,AX^* + t\,AXAX^*\;.$$

$$\tau_A\,(X^2) \neq X\,\tau_A\,(X) + \tau_A\,(X)\;X^* \;,$$

Then τ_A is not $* -derivation.$

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طبيعة الحل لمعادلة الاورام السرطانية المؤثرة

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المستخلص: في هذا البحث, نــــدرس حـــل معادلـــــة الاورام السلطانيـــة المؤثرة والتي تكون بالشكل $A^{*}XA + t\,AXA = Q$ ، حيث A يمثل مؤثر معلوم معرف في فضاء هلبرت H ويمثل العوامل المسببة والمساعده على السرطان (العامل الهرموني, عامل التغذية, العلاقة مابين العامل الهرموني وعامل التغذية, عامل الوراثه, الفيروسات ,...). و Q ايضاً مؤثر معلوم معرف في فضاء هلبرت H ويمثل نسب العوامل المساعده على السرطان(الجنس, العمر, التعرض للاشعاع, تعاطي المهدئات, الاجهاد النفسي, العوامل التي تضعف المناعة,...). t اي عدد موجب حقيقي يسمى عامل السيطرة و X هو المؤثر الغير معلوم ويمثل هذا المؤثر العلاج من حيث الكمية والنوعية للمصاب. وكذلك بينا طبيعة الحل لهذه المعادلة المؤثره لانواع خاصة من المؤثرات . بالاضافة الى سوف نقدم بعض الخصائص والملاحظات لانها مهمه جداً في الدراسة الحديثه في انظمة المعادلات التفاضلية الطبية (الحياتية).

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