

Testing Fuzzy Hypothesis with Application

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Abstract

The testing methods used to test hypothesis when the data about variables are uncertain, represent fuzzy testing. There are many kinds of fuzzy hypothesis, some of them about percentage of defectives, another related to the ratio of two populations mean or ratio of variances. The testing of fuzzy hypothesis may be simple hypothesis or compound hypothesis, here we introduce a test method for testing hypothesis about the percentage of defectives in certain electrical part produced by certain company, where this produced units are important and introduced with another units to complete the product. The testing of fuzzy hypothesis and everything related to this subject like acceptance and rejection, using procedure of Neyman person as a tool for testing fuzzy hypothesis. All derivations required finding testing rule of fuzzy hypothesis for the data under study. The aim of the research to introduce the statistical testing and statistical inference about parameters of population, and how to make a decision for rejecting or accepting the fuzzy hypothesis for the percentage of defective items.

Keywords:Fuzzy Testing, percentage of defectives, Neyman person, rejecting or accepting the fuzzy hypothesis.

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1. Theoretical Aspect

First of all we give some definitions;

Zimmerman (1988), defined a fuzzy group by^[13];

$$A = \{x, \mu_A(x) | x \in X\} \dots\dots\dots(1)$$

i.e each fuzzy set (A) is defined by it's element (x), and it's membership function ($\mu_A(x)$), where, ($0 \leq \mu_A(x) \leq 1$), and it have many different formula like, (NogoitandRaleeseu);

$$A = \sum_{i=1}^n \frac{\mu_A(x_i)}{x_i} \dots\dots\dots(2)$$

When (x) is infinite group;

$$A = \int_x \frac{\mu_A(x_i)}{x} dx \dots\dots\dots (3)$$

While the membership function which represent the degree of individual membership to group, the values of this degree depend on people to determine it, but there are some formulas used data to determine the function of membership, some of these functions are;

1. Triangular membership function

$$\mu(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & x < a \text{ or } x > c \end{cases} \dots\dots(4)$$

2. The second type is normal membership function;

$$\mu(x) = e^{-\left(\frac{x-a}{b}\right)^2} \quad b > 0 \quad \dots\dots\dots(5)$$

3. The third type is trapezoidal membership function denoted by;

$$\mu(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & x > d \end{cases} \dots\dots\dots(6)$$

Definition: The support of each fuzzy set (A) for $(x \in X)$ and $(\mu_A(x) > 0)$ defined by;

$$Sup(A) = \{x \in X; \mu_A(x) > 0\} \dots\dots\dots(7)$$

Definition ($\alpha - level$ set): The group of ($\alpha - level$) is ordinary group represent the element that belong to fuzzy set (A) with degree of at least (α), it is denoted by;

$$A_\alpha = \{x \in X; \mu_A(x) \geq \alpha\} \dots\dots\dots(8)$$

Is called strong ($\alpha - level$) set.

Definition: The fuzzy set (A) is called convex set if;

$$\mu_A[\lambda x_1 + (1 - \lambda)x_2] \geq Min[\mu_A(x_1), \mu_A(x_2)] \quad \forall \lambda \in [0,1], x_1, x_2 \in X \dots\dots(9)$$

Definition: The complement of membership function of fuzzy set (A) is;

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad \forall x \in X \quad [where Max \mu_A(x) = 1] \dots\dots\dots(10)$$

Definition (fuzzy random variable): The fuzzy random variable defined by (Shapiro 2008), is the random variable which have fuzzy probability space $(\tilde{\Omega}, \tilde{f}, \tilde{p})$, i.e, it is a results of application of;

$$\tilde{x}: \tilde{\Omega} \rightsquigarrow F(R^n) \dots\dots\dots(11)$$

Where $[F(R^n)]$ the set of all fuzzy number in (R^n) , also the sample space is fuzzy sample space which is denoted by (\tilde{x}) is partitioning to group (X) into sub fuzzy groups i.e;

$$[X = x \in R | f(x) > 0]$$

$$\sum_{\tilde{x} \in \tilde{X}} \mu_{\tilde{x}}(\bar{X}) = 1 \quad \forall x \in X \dots\dots\dots(12)$$

Definition (fuzzy valued random variable): The random variable with fuzzy valued is denoted by;

$$\tilde{x} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n) \dots\dots\dots(13)$$

With size (n) and related to function [f(x)], is a measurable function on;

$$\Omega \rightarrow \tilde{X}^n = \tilde{X} \times \tilde{X} \times \tilde{X} \times \dots \times \tilde{X} \dots\dots\dots(14)$$

Where (Ω) is fuzzy probability space with probability function;

$$f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n) = pr(\tilde{X} = \tilde{x}) = \int_{x^n} \prod_{i=1}^n \mu_{\tilde{x}_i}(x_i) f(x_i) dv(x_i) \dots(15)$$

Due to independence, it reduced to;

$$f(\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n) = pr(\tilde{X} = \tilde{x}) = \prod_{i=1}^n f(\tilde{x}_i)$$

$$f(\tilde{x}_i) = \int_{x^n} \mu_{\tilde{x}_i}(x_i) f(x_i) dv(x_i)$$

To test the fuzzy hypothesis;

$$H_0: \theta \in H(\theta)$$

$H(\theta)$: Membership function, we have [$H_0(\theta), H_1(\theta)$], two membership functions, we have to test;

$$H_0: \theta \in H_0(\theta)$$

$$H_1: \theta \in H_1(\theta)$$

According to random sample with fuzzy valued and at any test we have two types of error, type I error (which represent rejecting H_0 when H_0 is true) and type II error (accepting false hypothesis), type I error for fuzzy test function [$\tilde{\theta}(\tilde{X})$] is;

$$\alpha_{\tilde{\theta}} = E_0[\tilde{\theta}(\tilde{X})]$$

$$\beta_{\tilde{\theta}} = 1 - E_1[\tilde{\theta}(\tilde{X})]$$

Where $[E_j[\tilde{\Phi}(\tilde{X})]]$ represent the expected value for fuzzy test function $[\tilde{\Phi}(\tilde{X})]$ over the range of weighted fuzzy probability function, which is represented by;

$$\tilde{f}_j(\tilde{x}) = \prod_{i=1}^n \tilde{f}_j(\tilde{x}_i) = \prod_{i=1}^n \int_{\theta} H_j^*(\theta) f(\tilde{x}; \theta) d\theta \quad j = 0.1 \dots \dots (16)$$

According to Neyman Person lemma for testing fuzzy hypothesis, we have to testing hypothesis about any parameter (real or fuzzy), type I error may happen (which represent rejecting H_0 when H_0 is true), or type II error (which accepting false hypothesis), the probability of type I and type II for fuzzy hypothesis testing $[\tilde{\Phi}(\tilde{X})]$ is given by;

$$\alpha_{\tilde{\Phi}} = E_0[\tilde{\Phi}(\tilde{X})]$$

$$\beta_{\tilde{\Phi}} = 1 - E_1[\tilde{\Phi}(\tilde{X})]$$

Where $[E_j[\tilde{\Phi}(\tilde{X})]]$ a fuzzy probability function $[f(\tilde{x}, \theta)]$ we have $[\tilde{\Phi}(\tilde{X})]$ is a test fuzzy function which represent the probability of rejecting (H_0), the weighted fuzzy probability function at $[H_j(\theta)]$ is denoted by;

$$\left. \begin{aligned} \tilde{f}_j(\tilde{x}) &= \int_{\theta} H_j^*(\theta) f(\tilde{x}; \theta) d\theta \quad j = 0.1 \\ H_j^*(\theta) &= \frac{H_j(\theta)}{\int_{\theta} H_j(\theta) d\theta} \end{aligned} \right\} \dots (17)$$

Is a membership function, and;

$$\left. \begin{aligned} \int_{\theta} H_0(\theta) d\theta &< \infty \\ \int_{\theta} H_1(\theta) d\theta &< \infty \end{aligned} \right\} \dots \dots \dots (18)$$

2. Application

From the frequency distribution of percentage of defectives (p_i) in (330) lots taken form certain products of units in electrical company, we found that;

$$\bar{p} = \frac{18.5064}{330} = 0.05608$$

Also we found that (χ_{cal}^2);

$$\chi^2 = \sum_{i=1}^{10} \frac{(O_i - E_i)^2}{E_i} = 8.965$$

For the two estimated percentage of defectives given by company which are (0.045, 0.12, 0.94), we see that [$pr(\chi_{6 d.f}^2 = 8.965)$] as compared with tabulated value ($\chi_6^2 = 12.6$) for ($\alpha = 0.05$), we accept the hypothesis that the percentage defectives in this product is fuzzy number and can be represented by values (0.05608, 0.08705, 0.1206), so these estimated percentage of defectives can be used for fuzzy testing of hypothesis.

The membership function according to these three parameters is;

$$\mu_{\tilde{x}_1}(x) = \begin{cases} 0.05608 & x = 0 \\ 0.94392 & x = 1 \end{cases}$$

$$\mu_{\tilde{x}_{11}}(x) = \begin{cases} 0.94392 & x = 0 \\ 0.05608 & x = 1 \end{cases}$$

$$\left. \begin{aligned} H_j^*(\theta) &= \frac{H_j(\theta)}{\int_0^1 H_j(\theta) d\theta} \\ H_0^*(\theta) &= \frac{\theta^{A_0(1-\theta)}}{\int_0^1 \theta^{A_0(1-\theta)} d\theta} = \frac{\theta^{A_0(1-\theta)}}{Beta(A_0+1,2)} \end{aligned} \right\} \dots\dots(19)$$

Since;

$$A_0 = \frac{1-\theta_0}{\theta_0} \text{ then;}$$

$$H_0^*(\theta) = \theta^{A_0} (1 - \theta) \frac{1+\theta_0}{\theta_0^2} \dots\dots\dots (20)$$

We reject the hypothesis of one estimator of percentage of defective, here we have fuzzy estimator of (θ) as indicated, since;

$$f(x, \theta) = \theta^x (1 - \theta)^{1-x} \quad x = 0.1, \quad 0 < \theta < 1$$

Where (θ) is percentage of defective.

$$\left. \begin{aligned} H_0(\theta) &= \theta^{A_0}(1 - \theta) & \theta \in (0,1) \\ H_1(\theta) &= \theta(1 - \theta)^{A_1} & \theta \in (0,1) \end{aligned} \right\} \dots\dots\dots(21)$$

The fuzzy probability function;

$$f(\tilde{x}, \theta) = \sum_x \mu_{\tilde{x}_i}(x) f(x; \theta) \dots\dots\dots(22)$$

Which equal to;

$$f(\tilde{x}, \theta) = \begin{cases} 0.05608(1 - \theta) + 0.94392 \theta = 0.05608 + 0.88784\theta & \tilde{x} = \tilde{x}_1 \\ 0.94392(1 - \theta) + 0.05608 \theta = 0.94392 - 0.88784\theta & \tilde{x} = \tilde{x}_{11} \end{cases}$$

$$A_0 = \frac{1 - \theta_0}{\theta_0}, \quad A_1 = \frac{\theta_1}{1 - \theta_1}$$

$$H_0: \theta \approx \theta_0 = 0.05608$$

$$H_1: \theta \approx \theta_1 = 0.94392$$

$$A_0 = 16.8316, \quad A_1 = 16.8557$$

The normalized membership function for each $[H_0(\theta), H_1(\theta)]$ is denoted by;

$$H_j^*(\theta) = H_j(\theta) \left| \int_0^1 H_j(\theta) d\theta \right.$$

We have;

$$H_0^*(\theta) = \theta^{A_0}(1 - \theta) \frac{1 + \theta_0}{\theta_0^2}$$

$H_0^*(\theta)$ is the result of;

$$H_0^*(\theta) = H_0(\theta) \left| \int_0^1 H_0(\theta) d\theta \right. \dots\dots\dots (23)$$

Also $H_1^*(\theta)$ obtained after some steps;

$$H_1^*(\theta) = \theta(1 - \theta)^{A_1} \left(\frac{2 - \theta_1}{(1 - \theta_1)^2} \right) \dots\dots\dots(24)$$

Therefore the fuzzy weighted probability function is;

$$\tilde{f}_j(\tilde{x}) = \int_0^1 H_j^*(\theta) f(\tilde{x}; \theta) d\theta \quad \dots\dots\dots(25)$$

From equation (21), since the estimated parameters from data are, $(\theta_1, \theta_2, \theta_3)$, equal to $(0.05608, 0.08705, 0.1206)$, then, $(\mu_{\tilde{x}_1}(x) \text{ and } \mu_{\tilde{x}_{11}}(x))$ as shown above.

From equation (8), we can see (y_i) represents random variables, and values of (y_i) 's are iid, it can be represented by $[\tilde{f}_0(\tilde{x}_i)]$ under (H_0) and $[\tilde{f}_1(\tilde{x}_i)]$ under (H_1) . If (z_i) is random variable represent number of success and failure in the *p. d. f* of (y_i) , then the critical region for the test can be defined by;

$$C^* = (\tilde{x} | \sum_{i=1}^n z_i > C) \quad \dots\dots\dots (26)$$

Where $(z_i \sim \text{Bernollui})$ distribution with $[f_0(y_i)]$ under (H_0) and $[f_1(y_i)]$ under (H_1) .

The marginal fuzzy weighted probability function is;

$$H_1^*(\theta) = \frac{H_1(\theta)}{\int_0^1 H_1(\theta) d\theta} = \frac{\theta(1 - \theta)^{A_1}}{\text{Beta}(A_1 + 1, 2)}$$

But; $A_1 = \frac{\theta_1}{1 - \theta_1}$

$$H_1^*(\theta) = \theta(1 - \theta)^{A_1} \left(\frac{2 - \theta_1}{(1 - \theta_1)^2} \right)$$

According to above, the weighted fuzzy probability function $[\tilde{f}_j(\tilde{x})]$ is computed, after computing $[\tilde{f}_0(\tilde{x})]$ and $[\tilde{f}_1(\tilde{x})]$ from equation (17), we can apply Neyman – Person to find the most power critical region which is;

$$C = (\tilde{x} | \sum_{i=1}^n y_i \geq K) \quad \dots\dots\dots(27)$$

Where;

$$y_i = \ln \left(\frac{\tilde{f}_1(\tilde{x}_i)}{\tilde{f}_0(\tilde{x}_i)} \right) \quad \dots\dots\dots(28)$$

Since

($z_i \sim \text{Bernollui}(\theta)$) according to sample size (n) from the studied population, we can determine the most critical region (27).

If ($z_i \sim \text{Bernollui}(1, \theta)$), then ($\sum_{i=1}^n z_i \sim \text{Bernollui}(n, \theta)$), here it is denoted by ($\sum_{i=1}^n y_i$) and;

$$H_0: y_i \sim \text{Ber}(\theta = 0.056)$$

$$H_1: y_i \sim \text{Ber}(\theta = 0.944)$$

According to sample taken from product of size ($n = 60$), if ($\sum_{i=1}^n y_i = 3.3600 \cong 4$), if ($\sum_{i=1}^n y_i \leq 4$) accept this percentage defective according to Neyman – Person theorem, but when ($\sum_{i=1}^n y_i > 4$) reject (H_0) and we may search about the causes of deviation of quality of product from initial specification, is it due to material or to working reasons.

The estimated proportion of defectives in product is;

$$\hat{p} = \frac{30.844}{550} = 0.05608$$

The fuzzy percentage is not fixed, under fuzzy we have three categories (0.05608, 0.08705, 0.1206), since each produced unit follow Bernollui distribution;

$$f(x, \theta) = \theta^x (1 - \theta)^{1-x} \quad x = 0, 1 \quad 0 < \theta < 1$$

And ($\sum_{i=1}^n y_i$) have Binomial distribution with parameters (n, θ), then;

$$H_0(\theta) = \theta^{A_0} (1 - \theta)$$

$$H_1(\theta) = \theta (1 - \theta)^{A_1}$$

The values of membership function are;

$$H_0(\theta) = (0.045)^{21.222} (1 - 0.045) = (2.62105) 10^{-29} (0.005)$$

$$H_1(\theta) = 0.045 (0.995)^{0.13636} = 0.044718$$

That the percentage of defective is not fuzzy number, but it is fuzzy number represented by $(\theta_0 = 0.045, \theta_1 = 0.12)$. Here the percentage in product in the random sample (550 units) are;

$$H_0 = \theta \cong \theta_0 = 0.045$$

$$H_1 = \theta \cong \theta_1 = 0.955$$

Finally to reach a decision about fuzzy hypothesis, we must compute (fuzzy weighted probability function), equation (25), this gives;

$$\tilde{f}_0(\tilde{x}) = \begin{cases} 0.74 & \tilde{x} = \tilde{x}_1 \\ 0.26 & \tilde{x} = \tilde{x}_{11} \end{cases}$$

$$\tilde{f}_1(\tilde{x}) = \begin{cases} 0.26 & \tilde{x} = \tilde{x}_1 \\ 0.74 & \tilde{x} = \tilde{x}_{11} \end{cases}$$

Now compute equation (28) gives;

$$y_i = \begin{cases} -0.944662 & \tilde{x}_i = \tilde{x}_I \\ 0.94462 & \tilde{x}_i = \tilde{x}_{II} \end{cases}$$

According to Neyman – Person, the critical region (C), as shown in equation (27), we draw a sample of $(n = 10)$ from the studied pro

duct using certain level of significant $(\alpha = 0.05)$;

$$pr\left(\sum_{i=1}^n z_i \geq 7\right) = 1 - 0.9686 = 0.03414$$

That means the probability of rejecting the hypothesis when it is true is small (0.03414), also the power of the test ($p.o.t = 0.8976993 \cong 90\%$), this indicate that the probability of rejecting (H_0) is very high, i.e the percentage of defective in this product is not constant and cannot estimate by one value only, but by fuzzy values.

Conclusion

1. The testing of fuzzy hypothesis gives the researcher statistical inference about fuzzy parameters.
2. The power of fuzzy test which is the probability of rejecting false hypothesis is found 90 % which is high value.
3. We can estimate the number of defectives in the sample from using $[\mu = E(x) = n\hat{p}]$ also we can estimate the fuzzy number of outgoing quality level, which is necessary in testing large lots.
4. Studying fuzzy number and fuzzy hypothesis is good to recognition of uncertainty in the data, this lead to increasing interest of theoretical and practical aspects of fuzzy numbers and arithmetic.
5. Many fuzzy probability distributions have introduced, used in estimating and testing hypothesis where the estimation and testing are tools of statistics, these distribution are discrete or continuous.

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أختبار الفرضيات الضبابية: بحث تطبيقي

رابحة سليم كريم

المستخلص

تستخدم طرق الأختبار لغرض أختبار الفرضيات عندما تكون البيانات للمتغيرات غير مؤكدة، وتتمثل أختبار الضبابية. هنالك العديد من انواع الأختبارات الضبابية، قسم منها يتعلق بنسبة العيوب، والآخر يتعلق بنسبة مجتمعين او نسبة تبايناتها. أن اختبار الفرضيات الضبابية يمكن ان يكون اختبار بسيط او معقد، في بحثنا هذاسيتم اعتماد اختبار الفرضيات لنسبة العيوب لنوع محدد من الأجهزة الألكترونية منتج في إحدى الشركات الصناعية، وكل ما يتعلق بهذا الأختبار من رفض او قبول بأستخدام أسلوب (Neyman) كأداة للفرضيات الضبابية. تم التطرق لكافة العلاقات الرياضية التي تخص الموضوع في متن البحث، الذي يهدف الى توضيح للمعلمات والاستدلالات الأحصائية للمجتمع المدروس، وكيف يمكن صنع قرار لقبول او رفض الفرضيات الضبابية لنسب المعيب من الوحدات المنتجة.